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An intertemporal-optimizing general equilibrium model of exchange rates and external imbalances

Attapol Threemonkong
Iowa State University

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An intertemporal-optimizing general equilibrium model
of exchange rates and external imbalances

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by

Attapol Threemonkong

A Dissertation Submitted to the
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I. INTRODUCTION

After more than a decade of flexible exchange rates, there is still not a theoretical consensus on their determination. Recent work by Meese and Rogoff (1983) and Backus (1984) suggests that none of the leading models provide a complete explanation of observed exchanged rate movements. More strikingly, the emergence of a massive U.S. current account deficit has been one of the most troublesome macroeconomic developments of the 1980s. A number of studies have attributed the deficit to the decline in U.S. price competitiveness (associated with the appreciation of the dollar during the early 1980s), the relative strength of domestic growth in the United States, and the international debt situation. The relative importance of these factors in explaining the origin of the deficit varies across the studies, as do the roles these factors may play in resolving the deficit.

At a more fundamental level, it has been claimed that the origin of the deficit has been attributed to shifts in U.S. monetary and fiscal policies that reduced the national savings rate while raising real interest rates, domestic growth, and the dollar, relative to other countries. Several studies blame the U.S. fiscal expansion as the major causal factor; some even claim that the external deficit will persist until

the federal budget deficit is reduced. Others stress the importance of the U.S. monetary contraction in the early 1980s and exogenous shifts in international preferences for dollar assets.

While most of the literature on the deficit has a macroeconomic focus, a growing portion addresses the microeconomic factors underlying the deficit. To a certain extent, the literature reflects the increasing interest in the effects of productivity and technological competitiveness on external balance. Krugman and Hatsopoulos (1987), and Marston (1986) argue that the severity of the deterioration of the deficit was the result of macroeconomic factors combined with an underlying decline in the technological leadership of the United States and a slowdown in U.S. productivity growth relative to growth in other major industrial countries, especially Japan and Germany. In part, this catching up is to be expected as the U.S. economy matures (Japan, itself, may be slowing down relative to Korea).

While the literature focuses on macroeconomic and microeconomic causes, it has been traditionally done separately. Furthermore, the theoretical models are usually done under some presumed overall macroeconomic performance goal. On the other hand, using an intertemporal-optimizing general-equilibrium framework, many presumptions of the traditional international finance literature have been

overturned. Svenson and Razin (1983) shows that there is no particular reason to expect a permanent change in a nation's terms of trade to influence its current account balance. The reasoning has nothing to do with the price responsiveness of agents to relative price changes; rather, in an intertemporal context, the current account balance is viewed as a decision to save or dissave. Permanent relative price changes provide little incentive to intertemporally alter consumption/saving behavior.

Mussa (1984) shows that money neutrality implies nominal exchange rate neutrality; it is contradictory to assert that anticipated money supply changes are neutral but that changes in the nominal exchange rate have real effects. Another virtue of macro-models derived from individual optimizing behavior is that they are not subject to the Lucas (1976) critique; the behavioral rules of agents are consistent with the performance of the macroeconomic model. Lapan and Enders (1980) and Enders and Lapan (1983) show the importance of basing policy decisions on the behavioral rules of individuals rather than on some overall macroeconomic performance goal.

The study of the equivalence between tariffs and quotas is not new. Bhagwati and Srinivasan (1983) demonstrated the equivalence of tariffs and quotas in a partial-equilibrium framework. His argument was if a tariff were to be replaced by a quota equal to the import level associated with the

tariff, the quotas would lead to a domestic price that would exceed the landed, CIF price of the imported good by an implicit tariff that would equal the explicit tariff that the quota replaced and the real outcome would be identical. The only difference would be that in the case of the tariff the revenue would accrue to the government, whereas in the case of an equivalent quota an equal amount of windfall premia or "rents" would accrue to those receiving the import quotas.

This is illustrated in Figure 1.1, which shows a Marshallian diagram for price and quantity of an importable

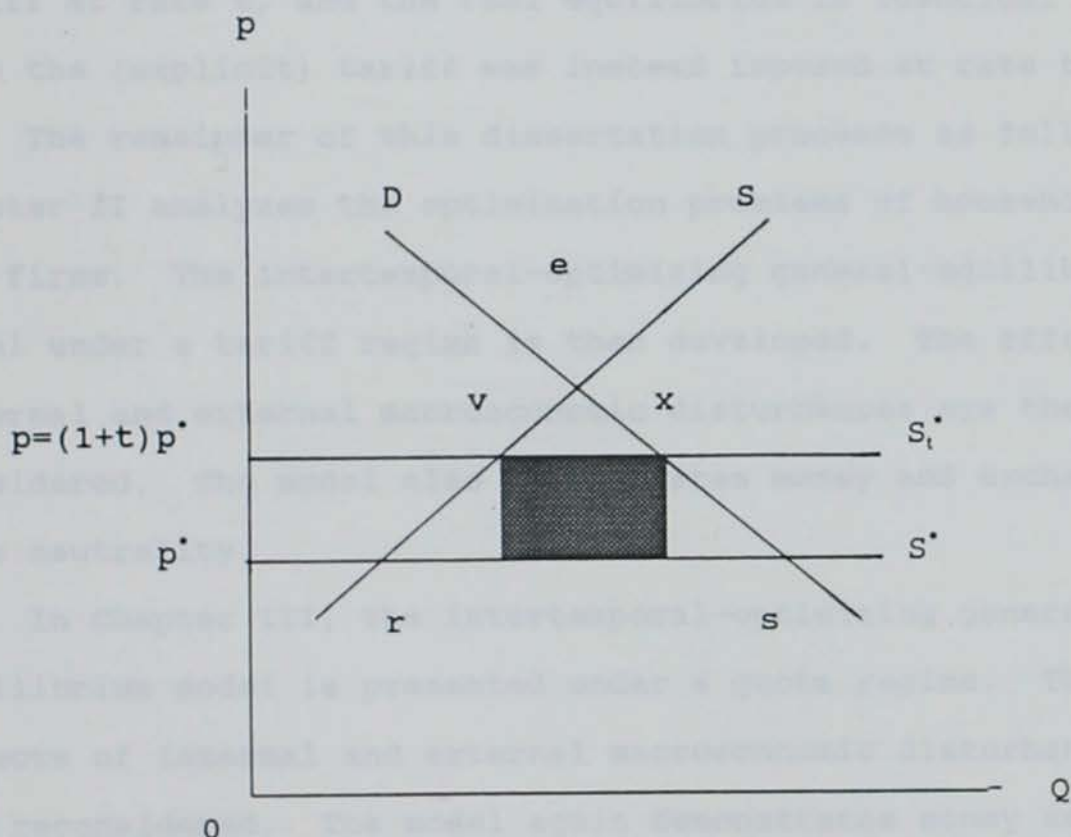


Figure 1.1. The equivalence of tariffs and quotas in a partial-equilibrium framework

good. D and S are the domestic demand and supply curves. S^* is the foreign supply curve, drawn for simplicity as a horizontal line at price p^* so that the economy is small in Samuelson's sense. With a tariff at rate t , the foreign supply curve S^* becomes S_t^* . Imports are therefore reduced from rs to vx . The domestic price p equals $(1+t)p^*$, and the government secures tariff revenue equal to the shaded area. Now replace the tariff by a quota at the tariff-associated import level vx . The market will evidently clear at domestic price p , whereas the CIF landed price for the imports is p^* . The difference between p and p^* then constitutes an implicit tariff at rate t , and the real equilibrium is identical to when the (explicit) tariff was instead imposed at rate t .

The remainder of this dissertation proceeds as follows. Chapter II analyzes the optimization problems of households and firms. The intertemporal-optimizing general-equilibrium model under a tariff regime is then developed. The effects of internal and external macroeconomic disturbances are then considered. The model also demonstrates money and exchange rate neutrality.

In Chapter III, the intertemporal-optimizing general-equilibrium model is presented under a quota regime. The effects of internal and external macroeconomic disturbances are reconsidered. The model again demonstrates money and exchange rate neutrality.

Chapter IV summarizes the major points and concludes this study.

The model region is a small open economy producing and consuming two goods, an agricultural good (denoted by a) and a non-agricultural good (denoted by n). The region is assumed to be as small as possible, it is assumed that:

1) The economy can be represented by a representative agent model in which the individual lives for two periods. For ease of exposition, it is assumed that the agent's objective function is such that production decisions can be analyzed separately from consumption decisions.

2) In the initial period of life, the individual must decide how much of each good to consume and how much to save (invest). Consumption decisions are made under conditions of perfect foresight as the individual has a well defined utility function.

3) As a producer, the agent must decide how much of each good which are not storable to produce in each period.

4) The institutional structure is such that currency is required to purchase commodities rather than barter. The individual is assumed to be a price taker in the domestic market for both goods and for the foreign market. Similarly, a domestic individual must first acquire the foreign currency.

II. AN INTERTEMPORAL-OPTIMIZING GENERAL-EQUILIBRIUM MODEL UNDER A TARIFF REGIME

Our model begins with a small open-economy producing and consuming two goods; an agricultural good (denoted by a) and a non-agricultural good (denoted by n). To keep matters as simple as possible, it is assumed that:

- 1) The economy can be portrayed by a representative agent model in which the individual lives for two periods. For ease of exposition, it is assumed that the agent's objective function is such that production decisions can be analyzed separately from consumption decisions.
- 2) In the initial period of life, the individual must decide how much of each good to consume and how much to save (dissave). Consumption decisions are made under conditions of perfect foresight so as to maximize a well defined utility function.
- 3) As a producer, the agent must decide how much of each good which are not storable to produce in each period.
- 4) The institutional structure is such that currency is required to purchase commodities--barter is assumed to be inefficient relative to exchanges using money. Domestic and foreigners wishing to buy domestic goods must use the domestic money. Similarly, a domestic wishing to import must first acquire the foreign currency.

5) Governments do not take an active role in the economy. Rather, the monetary authority simply supplies domestic currency to the banking system. For the time being, it is assumed that the fiscal authority imposes an import tariff on good a (which may be zero). Agents pay for all duties using the domestic currency. In each period, the government rebates the tariff revenue in a lump-sum fashion.

2.1 Supply

Within each period, production takes place along a concave production possibilities frontier. It is assumed that output of each good depends only on that period's domestic relative price of the good and a productivity factor:

$$(2.1) \quad a_i^f = A_i f^a[p_i]; \quad \frac{df^a}{dp_i} < 0$$

$$(2.2) \quad n_i^f = N_i f^n[p_i]; \quad \frac{df^n}{dp_i} > 0$$

$$(2.3) \quad p_i = \frac{p_{ni}}{p_{ai}}$$

where:

a_i^f (n_i^f) = production of good a (good n) in period i;

p_i = domestic relative price of good n in period i

$$(p_i = p_{ni}/p_{ai});$$

p_{ai} (p_{ni}) = domestic nominal price of good a (good n) in i;

$A_i (N_i)$ = multiplicative productivity term acting to shift the supply of good a (good n).

Since there are only two time periods ($i = 0, 1$) as a notational convenience the subscript zero can be dropped when it is unambiguous to do so.

Total production valued in terms of the domestic relative price of good a is:

$$(2.4) \quad y_i = a_i^s + p_i n_i^s$$

where:

y_i = total production valued in terms of the domestic relative price of the agricultural good.

Given that production takes place such that the absolute value of the slope of the production possibilities frontier equals the domestic relative price of good n:

$$(2.5) \quad \frac{dy_i}{dp_i} = n_i^s$$

It will be useful to define real interest rates. Since relative prices may change over time, real borrowing costs can be measured in terms of either good:

$$(2.6a) \quad (1 + r_a) = (1 + i) \left(\frac{p_a}{p_{a1}} \right)$$

$$(2.6b) \quad (1 + r_n) = (1 + i) \left(\frac{p_n}{p_{n1}} \right)$$

where:

r_i (r_n) = real interest rate measured in terms of good a (good n);

i = domestic nominal interest rate on borrowing (saving).

Equation (2.6) is the familiar definition of the real interest rate; one plus the real interest rate measured in terms of good a (good n) is equal to one plus the nominal interest rate divided by one plus the rate of price change of good a (good n). If the rate of price increase of good a (good n) exceeds the nominal interest rate, then r_i (r_n) is negative.

2.2 The Individual's Optimization Problem

Let the individual receive utility from the consumption of each good in each period of life:

$$(2.7) \quad u = u(a_0, n_0, a_1, n_1)$$

where:

a_i (n_i) = consumption of good a (good n) in period i ; and the function u is a well-behaved utility function.

In the beginning of each period, the individual must go to the financial market in order to obtain the currencies needed to purchase goods and pay import fees, and to save, dissave, or repay loans. The amount of domestic currency that the individual demands from the financial market is the sum of

purchases of agricultural goods from domestics plus the import fee on agricultural goods plus the purchases of non-agricultural goods from domestics:

$$(2.8) \quad m = p_a a' + p_n n + tep_a^* (a - a')$$

where:

m = number of units of domestic currency demanded by domestics;

e = domestic currency price of foreign exchange;

p_a = domestic currency price of good a ;

p_n = domestic currency price of good n ;

p_a^* = foreign currency price of good a ;

t = ad valorem tariff rate on good a so that tep_a^* = tariff per unit of a imported.

Note that the nation is an importer of the agricultural good so that consumption of good a equals the amount produced by domestic producers (a') plus imports ($a - a'$).

The individual must pay for imports using the foreign currency; since the foreign price of imports is p_a^* , the demand for the foreign currency is:

$$(2.9) \quad m^f = p_a^* (a - a')$$

The total amount of the two currencies demanded equals expenditures in the period. The individual budget constraint

states that expenditures plus saving equals disposable income. Within the general equilibrium framework presented here, the representative agent's disposable income is equal to the market value of sales [production] plus the transfer from the proceeds of the tariff revenue:

$$(2.10) \quad m + em^f = p_a a^s + p_n n^s - s + T$$

where:

s = saving;

T = transfer received from the government.

In the initial period, the agent knows that he/she will receive an income from sales equal to $p_a a^s + p_n n^s$ and a transfer from the government equal to T . Given this income, the individual decides how much to save (s) and how much to spend; total spending including the tariff duty is equal to $m + em^f$. At the end of the period, sellers of commodities hold $p_a a^s + p_n n^s$ units of domestic currency; the currency is deposited in the banking system. Any deposits in excess of expenditures earn the nominal interest rate i payable at the beginning of the next period.

In the beginning of the next period, then, the agent faces the three constraints:

$$(2.11) \quad m_1 = p_{a1} a_1^s + p_{n1} n_1 + t_1 e_1 p_{a1}^* (a_1 - a_1^s)$$

$$(2.12) \quad m_1^f = p_{a1}^* (a_1 - a_1^s)$$

$$(2.13) \quad m_1 + e_1 m_1^f = p_{a1} a_1^s + p_{n1} n_1^s + (1+i)s + T_1$$

In equation (2.11), domestic currency is used to purchase the quantities a_1^s and n_1 from domestics and to pay the period 1 tariff on imports. Foreign currency is used to purchase the quantity $a_1 - a_1^s$ of imports. Equation (2.13) states that the individual uses his/her gross income from sales plus saving and interest to acquire domestic and foreign currency. Since equation (2.13) holds with equality, the individual leaves no estate and pays all debts.

Thus, the individual's optimization problem is to select a , n , a_1 and n_1 so as to maximize the utility function of equation (2.7) subject to the constraints in equations (2.8)-(2.13). For presentation purposes, the problem can be simplified by forming the agent's lifetime budget constraint. First note that commodity arbitrage in the assumed absence of transport costs requires:

$$(2.14) \quad p_{n1} = e_1 p_{n1}^*$$

$$(2.15) \quad p_{a1} = (1 + t_i) e_1 p_{a1}^*$$

In equation (2.15), the full cost of importing good a is the foreign purchase price ($e_1 p_{a1}^*$) plus the tariff ($t_i e_1 p_{a1}^*$).

Combining equations (2.8) through (2.15) yields the individual's lifetime budget constraint:

$$(2.16) \quad p_a(a^s - a) + p_n(n^s - n) + T + \left[\frac{p_{a1}(a_1^s - a_1) + p_{n1}(n_1^s - n_1) + T_1}{(1+i)} \right] = 0$$

Selecting a , n , a_1 , and n_1 subject to the constraint of equation (2.16) so as to maximize the utility function of equation (2.7) yields the individual's demand functions.

2.3 Market Clearing

For the small country case under consideration, the goods market clears when desired imports (exports) equal actual imports (exports) and the arbitrage conditions in (2.14) and (2.15) hold. The domestic money market in period i clears when the total available stock of nominal money (M_i) is equal to the total demand for money. The foreign demand for the domestic currency (m_i^{*f}) is equal to the domestic currency value of their desired imports:

$$(2.17) \quad m_i^{*f} = p_{n1}(n_i^s - n_i)$$

where:

m_i^{*f} = the foreign demand for domestic currency;

$p_{n1}(n_i^s - n_i)$ = the domestic currency value of domestic exports (foreign imports).

The domestic money market clears when:

$$(2.18) \quad M_i = m_i + m_i^f$$

Finally, the assumption of perfect capital mobility links the domestic and the foreign credit markets. Letting i^* denote the foreign interest rate, interest rate arbitrage requires:

$$(2.19) \quad 1 + i = (1 + i^*) \frac{e_1}{e}$$

In the subsequent analysis, it will be useful to define the foreign real interest rate. Using a (*) to denote the foreign country counterpart of a domestic currency variable, the foreign real interest rates measured in terms of goods n and a are:

$$(2.20a) \quad (1 + r_n^*) \equiv (1 + i^*) \left(\frac{p_n^*}{p_{n1}^*} \right)$$

$$(2.20b) \quad (1 + r_a^*) \equiv (1 + i^*) \left(\frac{p_a^*}{p_{a1}^*} \right)$$

The real interest rate measured in terms of good n (good a) equals the nominal rate deflated by the rate of change in the price of good n (good a).

The small country assumption means that the domestic nation takes p_{n1}^* , p_{a1}^* and i^* (hence r_n^* and r_a^*) as exogenous.

2.4 General Equilibrium Solution

Given that the small-country is a price taker on the international commodity and capital markets, the solutions for the supply functions of equations (2.1) and (2.2) are obtained directly:

$$(2.21) \quad a_i^s = A_i f^a \left[\frac{p_i^*}{(1+t_i)} \right]$$

$$(2.22) \quad n_i^s = N_i f^n \left[\frac{p_i^*}{(1+t_i)} \right]$$

where: $p_i^* = p_{ni}^*/p_{ai}^*$ (foreign relative price of good n in period i).

The general equilibrium nature of the model is such that consumers view the output levels indicated by equations (2.21) - (2.22) as their endowments (or income levels).

Using the definitions (2.19) and (2.20), the agent's optimization problem can be written as:

$$(2.23) \quad \max u(a, n, a_1, n_1) + \lambda \left[w - (1+t)a - p^*n - \frac{(1+t_1)a_1}{(1+r_a^*)} - \frac{p^*n_1}{(1+r_n^*)} \right]$$

$$w \equiv (1+t)a^s + p^*n^s + T + \frac{(1+t_1)a_1^s}{(1+r_a^*)} + \frac{p^*n_1^s}{(1+r_n^*)} + \frac{T_1}{e_1 p_{a1}^* (1+r_a^*)}$$

where: λ is the Lagrangean multiplier; and w = discounted

value of the real income stream.

The solution to the optimization problem yields the demand functions:

$$(2.24a) \quad a = a \left[(1+t), p^*, \frac{(1+t_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

$$(2.24b) \quad n = n \left[(1+t), p^*, \frac{(1+t_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

$$(2.24c) \quad a_1 = a_1 \left[(1+t), p^*, \frac{(1+t_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

$$(2.24d) \quad n_1 = n_1 \left[(1+t), p^*, \frac{(1+t_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

where: w is defined in equation (2.23) and the values of the a_i^* , and n_i^* are given by equations (2.21) and (2.22), respectively.

Assuming all goods are normal means that the demand for each good in each period is positively related to w . Assuming gross substitutability implies an increase in the "own" price holding all other prices and w constant, reduces the demand for that good and increases the demand for the other three demands.

To fully complete the general equilibrium system, it is necessary to specify the government's budget constraint.

Given that government expenditure is zero and that its budget is balanced in every period, the transfers received by consumers are necessarily equal to the government's tariff revenues:

$$(2.25a) \quad T = t e p_a^* (a - a')$$

$$(2.25b) \quad T_1 = t_1 e_1 p_{a1}^* (a_1 - a_1')$$

Equations (2.21), (2.22), and (2.24) characterize the complete general equilibrium system.

2.4.1 A Specific Example

Let the individual have a utility function which is log-linear in the consumption of each good in each period of life. Specifically, let:

$$(2.26) \quad u = \theta \ln a_0 + (1-\theta) \ln n_0 + \frac{\theta}{(1+\rho)} \ln a_1 + \frac{(1-\theta)}{(1+\rho)} \ln n_1$$

where: θ is a share parameter such that $0 < \theta < 1$; and ρ is the subjective rate of time preference such that $\rho > 0$.

Maximizing equation (2.26) subject to the budget constraint yields the demand functions:

$$(2.27a) \quad a = \frac{\theta(1+\rho)}{(1+t)(2+\rho)} w$$

$$(2.27b) \quad a_1 = \frac{\theta(1+r_a^*)}{(1+t_1)(2+\rho)} w$$

$$(2.27c) \quad n = \frac{(1-\theta)(1+\rho)}{p^*(2+\rho)} w$$

$$(2.27d) \quad n_1 = \frac{(1-\theta)(1+r_n^*)}{p^*(2+\rho)} w$$

where: w is defined in equation (2.23).

Using the definition of w and substituting equations (2.24) and (2.25) into equation (2.27) yields the general equilibrium demand functions.

$$(2.28a) \quad a = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) \left[a^s + p^* n^s + \frac{p^* n_1^s}{1+r_n^*} + \frac{a_1^s}{1+r_a^*} \right]$$

$$(2.28b) \quad n = \frac{1}{\Delta} \left(\frac{1-\theta}{p^*} \right) \left(\frac{1+\rho}{2+\rho} \right) \left[a^s + p^* n^s + \frac{p^* n_1^s}{1+r_n^*} + \frac{a_1^s}{1+r_a^*} \right]$$

$$(2.28c) \quad a_1 = \frac{1}{\Delta} \left(\frac{\theta}{1+t_1} \right) \left(\frac{1+r_a^*}{2+\rho} \right) \left[a^s + p^* n^s + \frac{p^* n_1^s}{1+r_n^*} + \frac{a_1^s}{1+r_a^*} \right]$$

$$(2.28d) \quad n_1 = \frac{1}{\Delta} \left(\frac{1-\theta}{p^*} \right) \left(\frac{1+r_n^*}{2+\rho} \right) \left[a^s + p^* n^s + \frac{p^* n_1^s}{1+r_n^*} + \frac{a_1^s}{1+r_a^*} \right]$$

where:

$$\Delta = 1 - \left(\frac{\theta}{2+\rho} \right) \left(\frac{t(1+\rho)}{1+t} + \frac{t_1}{1+t_1} \right)$$

p^* = foreign relative price of good n ($p^* = p_n^*/p_a^*$)

and a^s, n^s, a_1^s, n_1^s are as defined in equations (2.21)

and (2.22).

2.5 Effects of Disturbances

One advantage of an intertemporal optimizing model is that the performance of the macro-economy is consistent with the behavioral rules of optimizing agents. As such, the model is not subject to the Lucas Critique (1976); it is possible to perform comparative static exercises using the general equilibrium solutions obtained in section 2.4 above. A second advantage of the approach taken here is that the savings/consumption function is consistent with the life-cycle hypothesis. Thus, the model is capable of analyzing the effects of anticipated future disturbances on current period behavior.

2.5.1 Productivity Changes

A wide variety of productivity shocks can be represented by appropriate changes in A_0 , A_1 , N_0 , and/or N_1 . For example, (i) a permanent productivity increase in the agricultural sector can be represented by equal increases in A_0 and A_1 ; (ii) an anticipated future increase by an increase in A_1 alone; and (iii) a current drought by a decline in A_0 . Sector-neutral technological change can be represented by proportional changes in A_i and N_i for $i = 0$ or 1 .

A key feature of the model is that productivity shocks are transmitted across sectors and across time through the actions of consumers on the demand side. Notice that the

discounted value of the real income stream (w) appears in the demand functions of equation set (2.24); a positive productivity shock in either sector in either period will increase all demands. In a general equilibrium model, outputs are the incomes of the households. Given that there is a diminishing marginal rate of substitution across commodities and across time, an increase in A_0 , N_0 , A_1 , or N_1 will increase the demand for each good in each period. The direct implication is that it is possible to observe changes in current period demand without any change in current period income. Individuals will lend (or borrow) in order to maintain the intertemporal marginal rate of substitution with the market interest rates. It is also worth making the obvious point that permanent shocks (such that $dA = dA_1$ or $dN = dN_1$) have larger consumption effects than temporary shocks (in which only one period's output changes).

On the supply-side, however, productivity shocks are not transmitted across sectors or across time. From examination of the general equilibrium solutions for a_i^* in equation (2.21), it is clear that production of good a in period i depends only on $p_i^*/(1+t_i)$ and A_i . Thus, productivity disturbances in the non-agricultural sector have no supply-side effects on the agricultural sector. In the same way, productivity changes in the agricultural sector have no supply-side effects on the non-agricultural sectors.

The effect of a productivity shock on the trade balance depends on whether the shock is temporary or permanent; temporary productivity shocks have a much stronger influence than permanent shocks. The sector in which the shock occurs is of little consequence for the trade balance. Since the nation exports good n and imports good a , the balance of trade measured in terms of the world price of good a is:

$$(2.29a) \quad tb = p^* (n^s - n) - (a - a^s)$$

$$(2.29b) \quad tb_1 = p_1^* (n_1^s - n_1) - (a_1 - a_1^s)$$

where: tb_1 = balance of trade measured in terms of the world price of the agricultural good.

As an aid to understanding the model, substitute equations (2.25a) and (2.25b) and the definitions of the real interest rates into the individual's lifetime budget constraint (equation (2.16)) to obtain:

$$(2.30) \quad (a^s - a) + p^* (n^s - n) + \frac{(a_1^s - a_1)}{(1 + r_a^*)} + \frac{p^* (n_1^s - n_1)}{(1 + r_n^*)} = 0$$

For the economy as a whole, the tariff does not involve a direct income effect; the discounted value of the lifetime consumption stream must equal the discounted value of the

lifetime income stream. The role of the tariff is to distort prices, hence consumption and production decisions. Combining equations (2.29) and (2.30) reveals a fundamental property of the model; the discounted value of the trade balance must be zero:

$$(2.31) \quad tb + \frac{tb_1}{(1+r_a^*)} = 0$$

Equation (2.30) demonstrates that over the course of the agent's lifetime, spending cannot exceed income. As a result, equation (2.31) demonstrates that the economy cannot experience a perpetual external deficit or surplus. Any excess of spending over income in the initial period must be repaid in the subsequent period.

Any changes in productivity will affect the current value of the trade balance only to the extent that individuals are induced to save or dissave.

Ruling out inferior goods, demand responds positively to a productivity increase in either sector in either period. For the trade balance, the results can be summarized as follows:

i) Initial period productivity increases which are temporary (i.e., only $da > 0$ or $dN > 0$) have a positive effect on the trade balance. Individuals attempt to "smooth-out" the effects of an increase in current income by saving. As a

nation, the increase in saving is equivalent to a balance of trade surplus. In the subsequent period, however, agents dissave by generating an external deficit.

ii) Period 1 productivity gains (i.e., only $dA_1 > 0$ or $dN_1 > 0$) have a negative effect on the initial period's trade balance. The increased productivity induces individuals to borrow against their future income in order to finance current consumption. As current expenditures rise relative to current income, the trade balance deteriorates. In period 1, however, individuals repay their loans; the trade balance for period 1 shows a surplus.

iii) Permanent productivity changes have an ambiguous effect on the trade balance; if income levels in both periods increase, there is no presumption as to whether individuals will save or dissave. Tables 2.1 - 2.3 present the effects of changes in A_0 , A_1 , N_0 , and N_1 , on current period demands and the trade balance for the case of the specific utility function given by equation (2.26). Evaluating at $t = t_1 = 0$, a permanent productivity shock in good a will have no effect on the trade balance if $1+r_n^* = 1+\rho$ and a permanent productivity shock to good n will have no effect on the trade balance if $1+r_n^* = 1+\rho$. Simply put, if the subjective rate of time preference is equal to the real interest rate, there is no incentive to intertemporally transfer additional income into the subsequent period when both income levels increase by the

Table 2.1. Consumption effects of productivity shocks on current agricultural goods under a tariff regime

$$0 < \frac{da}{dA} = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) < 1$$

$$0 < \frac{da}{dA_1} = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1}{1+r_a^*} \right) < 1$$

$$\frac{da}{dA} + \frac{da}{dA_1} = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{2+r_a^*}{1+r_a^*} \right) > 0$$

$$\frac{da}{dN} = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) p^* > 0$$

$$\frac{da}{dN_1} = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) \frac{p^*}{1+r_n^*} > 0$$

$$\frac{da}{dN} + \frac{da}{dN_1} = \frac{1}{\Delta} \left(\frac{\theta}{1+t} \right) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{2+r_n^*}{1+r_n^*} \right) p^* > 0$$

where: $\Delta = 1 - \left(\frac{\theta}{2+\rho} \right) \left(\frac{t(1+\rho)}{1+t} + \frac{t_1}{1+t_1} \right)$

Table 2.2. Consumption effects of productivity shocks on current non-agricultural goods under a tariff regime

$$\frac{dn}{dA} = \frac{1}{\Delta} \frac{1}{p^*} (1-\theta) \left(\frac{1+\rho}{2+\rho} \right) > 0$$

$$\frac{dn}{dA_1} = \frac{1}{\Delta} \frac{1}{p^*} (1-\theta) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1}{1+r_a^*} \right) > 0$$

$$\frac{dn}{dA} + \frac{dn}{dA_1} = \frac{1}{\Delta} \frac{1}{p^*} (1-\theta) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{2+r_a^*}{1+r_a^*} \right) > 0$$

$$0 < \frac{dn}{dN} = \frac{1}{\Delta} (1-\theta) \left(\frac{1+\rho}{2+\rho} \right) < 1$$

$$0 < \frac{dn}{dN_1} = \frac{1}{\Delta} (1-\theta) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1}{1+r_n^*} \right) < 1$$

$$\frac{dn}{dN} + \frac{dn}{dN_1} = \frac{1}{\Delta} (1-\theta) \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{2+r_n^*}{1+r_n^*} \right) > 0$$

where: $\Delta = 1 - \left(\frac{\theta}{2+\rho} \right) \left(\frac{t(1+\rho)}{1+t} + \frac{t_1}{1+t_1} \right)$

Table 2.3. Consumption effects of productivity shocks on current trade balance under a tariff regime

$$\frac{dtb}{dA} = 1 - \frac{1}{\Delta} \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1+t(1-\theta)}{1+t} \right) > 0$$

$$\frac{dtb}{dA_1} = - \frac{1}{\Delta} \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1}{1+r_a^*} \right) \left(\frac{1+t(1-\theta)}{1+t} \right) < 0$$

$$\frac{dtb}{dA} + \frac{dtb}{dA_1} = 1 - \frac{1}{\Delta} \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{2+r_a^*}{1+r_a^*} \right) \left(\frac{1+t(1-\theta)}{1+t} \right) = ?$$

$$\frac{dtb}{dN} = p^* \left[1 - \frac{1}{\Delta} \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1+t(1-\theta)}{1+t} \right) \right] > 0$$

$$\frac{dtb}{dN_1} = - \frac{p^*}{\Delta} \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{1}{1+r_n^*} \right) < 0$$

$$\frac{dtb}{dN} + \frac{dtb}{dN_1} = p^* \left[1 - \frac{1}{\Delta} \left(\frac{1+\rho}{2+\rho} \right) \left(\frac{2+r_n^*}{1+r_n^*} \right) \left(\frac{1+t(1-\theta)}{1+t} \right) \right] = ?$$

where: $\Delta = 1 - \left(\frac{\theta}{2+\rho} \right) \left(\frac{t(1+\rho)}{1+t} + \frac{t_1}{1+t_1} \right)$

same amount. For a nation with a high rate of time preference ($\rho > r_1^*$ or $\rho > r_n^*$), a permanent productivity increase will be associated with an initial period deficit and a surplus in period 1.

The key point is that there is no simple relationship between productivity and the trade balance. Permanent productivity changes in either the import or the export sectors may generate trade surpluses or deficits. Moreover, current period productivity may be high yet the trade balance may exhibit a deficit if future incomes are expected to be higher than current income. Studies which forecast the direction of the trade balance without decomposing the movements in income levels into their permanent versus temporary components are bound to be misleading.

2.5.2 External Disturbances

The small open economy takes p^* , r_1^* , and r_n^* as given; for a given tariff schedule, relative prices can change only through disturbances on international markets. As was the case for productivity shocks, the effects of temporary external shocks are sometimes quite different from those of permanent shocks.

2.5.2.1 Permanent Changes in the Terms of Trade

Consider first the consequences of a change in p^* for given

values of r_s^* and r_n^* . From the definition in equation set (2.22):

$$(2.32) \quad p^* = \frac{p_n^*}{p_a^*}$$

Since the nation imports agricultural goods and exports good n, an increase in p^* represents an improvement in the nation's terms of trade. Given the foreign nominal interest rate (i^*), equations (2.20a) and (2.20b) demonstrate that unchanged values of r_s^* and r_n^* necessitate there be no change in the ratios p_s^*/p_{s1}^* and p_n^*/p_{n1}^* . Thus, an increase in p^* holding r_s^* and r_n^* constant must be interpreted as a permanent improvement in the nation's terms of trade (both p_n^*/p_s^* and p_{n1}^*/p_{s1}^* rise by equal amounts).

The supply functions [equations (2.21) and (2.22)] show that the output of each good is positively related to "own" price; thus, a permanent increase in the terms of trade will increase the supply of good n and decrease the supply of good a in each period. On the supply side, then, the overall effect of the permanent improvement in the nation's terms of trade is to increase the quantity of good n and decrease the quantity of good a marketed in each period. Without imposing additional structure on the model, little can be said about the demand functions other than "own-price" effects are

negative. For the specific log-linear utility function, however, the effects of a change in relative prices can be calculated directly from equation (2.27). The effects of an increase in p^* are listed in the first row of Tables 2.4 - 2.8 where, for simplicity, all total derivatives are evaluated at the point $t = t_1 = 0$. As seen in the Tables, a permanent increase in the relative price of good n reduces the demand for non-agricultural goods and increases the demand for agricultural goods in each period.

It is crucial to note that there is no presumption that an improvement in a nation's terms of trade will improve a nation's trade balance. It is true that a reduction in the demand for good n coupled with an increase in supply results in an overall increase in the export supply function. However, the increased demand for the agricultural good coupled with a reduction in supply results in an increase in import demand. In each period exports and imports increase. Rewriting equation (2.29a):

$$(2.33) \quad tb = a' + p^*n' - (a + p^*n)$$

The trade balance is the difference between total domestic production ($a' + p^*n'$) and total domestic expenditures ($a + p^*n$). The intuitive explanation of the ambiguous effect of a permanent change in p^* on the trade balance is that there

Table 2.4. Effects of relative price changes on current agricultural goods under a tariff regime

$$\frac{da}{dp^*} = \theta \left(\frac{1+\rho}{2+\rho} \right) \left(n^s + \frac{n_1^s}{1+r_n^*} \right) > 0$$

$$\frac{da}{dr_n^*} = -\theta \left(\frac{1+\rho}{2+\rho} \right) \frac{p^* n_1^s}{(1+r_n^*)^2} < 0$$

$$\frac{da}{dr_a^*} = -\theta \left(\frac{1+\rho}{2+\rho} \right) \frac{a_1^s}{(1+r_a^*)^2} < 0$$

Table 2.5. Effects of relative price changes on current
non-agricultural goods under a tariff regime

$$\frac{dn}{dp^*} = -(1-\theta) \left(\frac{1+\rho}{2+\rho} \right) \left(a^j + \frac{a_1^j}{1+r_a^*} \right) \left(\frac{1}{p^*} \right)^2 < 0$$

$$\frac{dn}{dr_n^*} = - \left(\frac{1-\theta}{p^*} \right) \left(\frac{1+\rho}{2+\rho} \right) \frac{p^* n_1^j}{(1+r_n^*)^2} < 0$$

$$\frac{dn}{dr_a^*} = - \left(\frac{1-\theta}{p^*} \right) \left(\frac{1+\rho}{2+\rho} \right) \frac{a_1^j}{(1+r_a^*)^2} < 0$$

Table 2.6. Effects of relative price changes on future agricultural goods under a tariff regime

$$\frac{da_1}{dp^*} = \theta \left(\frac{1+r_a^*}{2+\rho} \right) \left(n^s + \frac{n_1^s}{1+r_n^*} \right) > 0$$

$$\frac{da_1}{dr_n^*} = -\theta \left(\frac{1+r_a^*}{2+\rho} \right) \frac{p^* n_1^s}{(1+r_n^*)^2} < 0$$

$$\frac{da_1}{dr_a^*} = \left(\frac{\theta}{2+\rho} \right) \left(a^s + p^* n^s + \frac{p^* n_1^s}{1+r_n^*} \right) > 0$$

Table 2.7. Effects of relative price changes on future
non-agricultural goods under a tariff regime

$$\frac{dn_1}{dp^*} = -\frac{(1-\theta)}{p^{*1}} \left(\frac{1+r_n^*}{2+\rho} \right) \left(a^s + \frac{a_1^s}{1+r_a^*} \right) < 0$$

$$\frac{dn_1}{dr_n^*} = \frac{1}{p^*} \left(\frac{1-\theta}{2+\rho} \right) \left(a^s + \frac{a_1^s}{1+r_a^*} + p^* n^s \right) > 0$$

$$\frac{dn_1}{dr_a^*} = -\left(\frac{1-\theta}{p^*} \right) \left(\frac{1+r_n^*}{2+\rho} \right) \frac{a_1^s}{(1+r_a^*)^2} < 0$$

Table 2.8. Effects of relative price changes on current trade balance under a tariff regime

$$(2.74) \quad \frac{d(tb)}{dp^*} = n^s - \frac{1+\rho}{2+\rho} \left[n^s + \frac{n_1^s}{1+r_n^*} \right] = ?$$

$$\frac{d(tb)}{dr_n^*} = \left(\frac{1+\rho}{2+\rho} \right) \frac{p^* n_1^s}{(1+r_n^*)^2} > 0$$

$$\frac{d(tb)}{dr_a^*} = \left(\frac{1+\rho}{2+\rho} \right) \frac{a_1^s}{(1+r_a^*)^2} > 0$$

is no particular incentive for agents to intertemporally relocate expenditures in response to a permanent change in relative prices. Formally, differentiating equation (2.34) with respect to the terms of trade:

$$(2.34) \quad \frac{d(tb)}{dp^*} = (n^s - n) - p^* \frac{dn}{dp^*} - \frac{da}{dp^*}$$

$$= (n^s - n) - n\epsilon_n - \frac{a}{p^*} \epsilon_a$$

where: ϵ_x = total price elasticity of demand for good x.

Thus, $\epsilon_n = (p/n)(dn/dp) < 0$; $\epsilon_a = (p/a)(da/dp) > 0$. For simplicity, equation (2.34) is evaluated at $t = t_1 = 0$; thus, in deriving (2.34) it is possible to set $p = p^*$ and to use equation (2.5) directly.

In general, the sign of equation (2.34) is ambiguous. For a nation exporting good n, $(n^s - n)$ is positive; the increase in p^* increases the value of exports for unchanged levels of production and demand. The expression $(-n\epsilon_n)$ is also positive; the greater the elasticity of demand for n the greater is the substitution out of the current period demand for good n and the greater the improvement in the trade balance. However, $(-a/p^*)\epsilon_a$ is negative; the greater this elasticity of demand, the greater is the substitution into the current period demand for the imported good.

For the specific utility function, substitute the entries

listed in Tables 2.4 - 2.5 into equation (2.34) to obtain:

$$(2.35) \quad \frac{d(tb)}{dp^*} = n^s - \frac{1+\rho}{2+\rho} \left[n^s + \frac{n_1^s}{1+r_n^*} \right]$$

To interpret, the sign of the trade balance depends on the pattern of production (n^s versus n_1^s), the rate of time preference (ρ), and the real interest rate. Since the nation exports good n , an improvement in the terms of trade causes a real income gain in each period; the agent must choose how to allocate this gain across time. A large value of n_1^s relative to n^s implies that the preponderance of the real income gain is associated with future production; the agent will tend to increase present consumption relative to present income. A large value of ρ relative to r_n^* implies a preference for current period consumption; the agent will choose to allocate a large proportion of a given increase in real income to present expenditure.

2.5.2.2 Interest Rate Changes

It is useful to separate a change in relative commodity prices from a change in the cost of borrowing. Given foreign prices (i.e., p_1^* , p_n^* , p_{n1}^* , and p_{nn}^*), a pure increase in the cost of borrowing is represented by equiproportionate increases in $1+i^*$, $1+r_1^*$, and $1+r_n^*$. On the supply side, production levels remain unaltered. Assuming that commodities are gross substitutes in the

consumers' budget, the increase in real interest rates induces a substitution away from present consumption towards future consumption. In this circumstance, the trade balance improves since production is unaltered, current sales of good a increase, and current period expenditures on good a and n decline.

2.5.2.3 Temporary Price Changes Temporary changes in commodity prices can be viewed as a combination of a real interest rate change and a change in the terms of trade. For example, a temporary increase in the relative price of good n (i.e., an increase in p^* without any change in p_1^*) does not involve a change in $(1+i^*)$ or $(1+r_n^*)$ but r_n^* changes such that $p^*/(1+r_n^*)$ remains constant. An increase in the period-one price of good a (p_{a1}^*) can be represented by a reduction in p_1^* and $(1+r_n^*)$ for constant values of p^* , $(1+i^*)$, and $(1+r_n^*)$. For a temporary increase in the relative price of good n, the current period production of good n increases at the expense of good a, and--if commodities are gross substitutes--the demand for good n declines and the demand for good a increases. The second and third rows of Tables 2.4 - 2.8 give the effects of pure changes in interest rates on commodity demands and the trade balance for the specific log-linear utility function.

2.6 Money and Exchange Rate Neutrality

An important feature of the model is that it exhibits exchange rate/money neutrality. Notice that the behavior of the real variables in the system could be determined without reference to the nominal exchange rate or to the money supply. On the other hand, prices and the exchange rate are determined by movements in the real variables in the system.

To solve for the nominal variables, substitute the domestic demand for domestic money (equation (2.8)) and the foreign demand for domestic money (equation (2.17)) into the money market clearing condition (equation (2.18)). In the initial period, money market equilibrium entails:

$$(2.36) \quad M = p_a a^s + p_n n + t e p_a^* (a - a^s) + p_n (n^s - n) \\ = e p_a^* [a^s + p^* n^s + t a]$$

where: a is determined as in equation (2.24). For the case of specific log-linear utility function:

$$(2.37) \quad M = e p_a^* \left[a^s + p^* n^s + \frac{t}{1+t} \frac{\theta(1+\rho)}{2+\rho} \frac{1}{\Delta} \left(a^s + p^* n^s + \frac{p^* n_1^s}{1+r_n^*} + \frac{a_1^s}{1+r_a^*} \right) \right]$$

Given that a^s , $p^* n^s$ and a are not affected by a money supply change, the nominal exchange rate is directly

proportional to the domestic money supply and indirectly proportional to p_a^* , p_{a1}^* , p_n^* , and p_{n1}^* (if these four prices all rise by the proportion ω , p^* , r_a^* , and r_n^* remain unaltered while the exchange rate falls in the proportion $\omega^{0.5}$). Additionally, the exchange rate depends on current and future values of the real variables in the system. An increase in current output of either good will act to appreciate the domestic currency as increased transactions increase the demand for the domestic currency. The value of the domestic currency will be decreasing in r_a^* . Also, as long as $t \neq 0$, the demand for domestic money depends on the tariff revenue from imports; increases in future output levels act to increase the current demand for good a and so act to appreciate the domestic currency.

The key point is that the exchange rate is completely endogenous; co-movements between the nominal exchange rate and other variables in the system cannot be attributed to the exchange rate itself. Moreover, there is no simple relationship between the exchange rate and the other endogenous variables in the system. For example, positive productivity shocks to a_1^* or n_1^* will worsen the trade balance in the initial period and appreciate the domestic currency. In no sense can it be said that the decline in the trade balance is due to movements in the nominal exchange rate. Although future productivity shocks are associated with

currency appreciation and a trade balance deficit, an increase in current productivity will be associated with currency appreciation (de/da' and $de/dn' < 0$) and a trade surplus.

The small open economy considered in chapter 2 is now analyzed under a quota regime. The economy's factor endowments and technology still hold. However, the institutional structure is now assumed to be such that the government issues import licenses; each license allows the holder to import one unit of the agricultural good. Let the symbol q denote the quantity of licenses issued during time period t . As long as the government captures the quota revenue, collects the revenue in domestic currency, and rebates the proceeds to the public in lump-sum fashion, there is vector of prices $(p_t, 1)$ which is equivalent to any arbitrary tariff vector $(\tau_t, 1)$.

To show this equivalence, assume that in each period the government sells q import licenses at the competitively determined price p_L . Each license enables the holder to purchase good a at the world price p_W and to resell the good at the domestic price p_D . Competition ensures that the domestic price of good a differs from the world price by the exact amount of the price of the import license:

$$(2.3) \quad p_D = p_W + p_L$$

or:

$$(2.3') \quad \frac{p_D}{p_W} = 1 + \frac{p_L}{p_W}$$

III. AN INTERTEMPORAL-OPTIMIZING GENERAL-EQUILIBRIUM MODEL UNDER A QUOTA REGIME

The small open-economy analyzed in chapter 2 is now examined under a quota regime. The economy's first four characteristics still hold. However, the institutional structure is now assumed to be such that the government issues import licenses; each license allows the holder to import one unit of the agricultural good. Let the symbol q_i denote the quantity of licenses issued during time period i . As long as the government captures the quota revenue, collects the revenue in domestic currency, and rebates the proceeds to the public in lump-sum fashion, there is vector of quotas $[q_0, q_1]$ which is equivalent to any arbitrary tariff vector $[t_0, t_1]$.

To show this equivalence, assume that in each period the government sells q_i import licenses at the competitively determined price p_{qi} . Each license enables the holder to purchase good a at the world price $e_i p_{ai}^*$ and to resell the good at the domestic price p_{ai} . Competition ensures that the domestic price of good a differs from the world price by the exact amount of the price of the import license:

$$(3.1) \quad p_{ai} = p_{qi} + e_i p_{ai}^*$$

or:

$$(3.2) \quad \frac{p_{ai}}{e_i p_{ai}^*} = 1 + g_i$$

where: g_i is defined to be $p_{qi}/e_i p_{ai}^*$.

The expression g_i is a measure of the "price-gap" between the domestic and the foreign price that is introduced by the quota. If $g_0 = t_0$ and $g_1 = t_1$, it is straightforward to show that the quota regime and the tariff regime will be identical in all respects.

Since the quota affects the price of good a only, the domestic relative price of good n in period i ($p_i \equiv p_{ni}/p_{ai}$) is:

$$(3.3) \quad p_i = \frac{p_i^*}{(1+g_i)}$$

3.1 Supply

Similar to the tariff-regime case, output of each good depends only on that period's domestic relative price of the good and a productivity factor. Therefore, the supply equations can also be represented by substituting equation (3.3) into equation (2.1) and (2.2):

$$(3.4) \quad a_i^s = A_i f^a \left[\frac{p_i^*}{(1+g_i)} \right]$$

$$(3.5) \quad n_i^s = N_i f^n \left[\frac{p_i^*}{(1+g_i)} \right]$$

Given the equality between t_i and g_i , supplies of each

good under the two regimes will be equal in each period.

From the definitions of the real interest rate (equations (2.6a) and (2.6b)), the interest rate parity condition, and the definitions of real foreign interest rates (equations (2.20a) and (2.20b)), it is easily verified that domestic real interest rates are tied to foreign real interest rates by the relationship:

$$(3.6a) \quad 1+r_a = \frac{(1+r_a^*)(1+g)}{(1+g_1)}$$

$$(3.6b) \quad 1+r_n = 1+r_n^*$$

Comparing equations (2.20a), (2.20b) and (2.23) to those above shows that if $g = t$ and $g_1 = t_1$, the values of real interest rates in the tariff regime will be equal to those in prevailing the quota regime.

3.2 The Individual's Optimization Problem

Having established the equivalence of the two regimes on the supply-side, it can be shown that if $g = t$ and $g_1 = t_1$ the consumer's intertemporal budget constraint is also invariant to the choice of tariffs versus quotas.

Again, we let the individual receive utility from the consumption of each good in each period of life:

$$(3.7) \quad u = u(a_0, n_0, a_1, n_1)$$

In the initial period, the amount of domestic currency that the individual demands from the financial market is the sum of purchases of agricultural goods from domestics plus the cost of import licenses on agricultural goods plus the purchases of non-agricultural goods from domestics:

$$(3.8) \quad m = p_a a^s + p_n n + g e p_a^* (a - a^s)$$

Similarly, the individual demand for domestic currency in the next period is

$$(3.9) \quad m_1 = p_{a1} a_1^s + p_{n1} n_1 + g_1 e_1 p_{a1}^* (a_1 - a_1^s)$$

Replace equations (2.8) and (2.11)--the individual's demands for domestic currency under that tariff regime--with (3.8) and (3.9) and use equations (2.9), (2.10), (2.12), and (2.13) to solve for the intertemporal budget constraint. The result is identical to equation (2.16).

The agent's optimization problem can be written as:

$$(3.10) \quad \max u(a, n, a_1, n_1) + \lambda \left[w - (1+g)a - p^* n - \frac{(1+g_1)a_1}{(1+r_a^*)} - \frac{p^* n_1}{(1+r_n^*)} \right]$$

$$w \equiv (1+g)a^s + p^* n^s + \frac{T}{e p_a^*} + \frac{(1+g_1)a_1^s}{(1+r_a^*)} + \frac{p^* n_1^s}{(1+r_n^*)} + \frac{T_1}{e_1 p_{a1}^* (1+r_a^*)}$$

and

$$(3.11) \quad T = g e p_a^* (a - a')$$

$$(3.12) \quad T_1 = g_1 e_1 p_{a1}^* (a_1 - a_1')$$

Since the tariff and quota revenues are equal, the following solution to the consumer's optimization problem is identical to that of equation (2.24)

$$(3.13a) \quad a = a \left[(1+g), p^*, \frac{(1+g_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

$$(3.13b) \quad n = n \left[(1+g), p^*, \frac{(1+g_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

$$(3.13c) \quad a_1 = a_1 \left[(1+g), p^*, \frac{(1+g_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

$$(3.13d) \quad n_1 = n_1 \left[(1+g), p^*, \frac{(1+g_1)}{(1+r_a^*)}, \frac{p^*}{(1+r_n^*)}, w \right]$$

where: w is defined in equation (3.10) and the values of the a_1' , and n_1' are given by equations (3.4) and (3.5), respectively.

The implication is the government can use its two instruments $[q, q_1]$ to achieve the price gaps $[g, g_1]$. Thus,

for any given tariff schedule $[t, t_1]$, the government can select $[q, q_1]$ to perfectly mimic the tariff regime. Moreover, the values for q and q_1 are equal to the quantity of imports prevailing under the tariff regime. Clearly, the equivalence works in both directions; a quota regime can be converted into an equivalent tariff regime by selecting tariff rates such that $t = g$ and $t_1 = g_1$.

However, it is incorrect to assert that the tariff t_0 is equivalent to the price-gap g_0 and that the tariff t_1 is equivalent to the price-gap g_1 . Rather, the tariff vector $[t, t_1]$ is equivalent to the price-gap vector $[g, g_1]$. In order to replace a quota regime with an equivalent regime, it is not sufficient to select a tariff rate t_0 equal to the price-gap g_0 . As shown in equation 2.24 and 3.13, behavior in the initial period, for example, depends on the tariff rate (or price-gap) in both periods. Replacing quotas (and other non-tariff trade barriers) with equivalent tariffs must be conducted in an intertemporal context such that $t = g$ and $t_1 = g_1$.

3.3 Market Clearing and General Equilibrium Solution

For the quota-regime case, the market clearing conditions discussed in Section 2.3 are still valid.

3.3.1 A Specific Example

Similar to the tariff case, let the individual have a

utility function which is log-linear in the consumption of each good in each period of life. Specifically, let:

$$(3.14) \quad u = \theta \ln a_0 + (1-\theta) \ln n_0 + \frac{\theta}{(1+\rho)} \ln a_1 + \frac{(1-\theta)}{(1+\rho)} \ln n_1$$

where: θ is a share parameter such that $0 < \theta < 1$; and ρ is the subjective rate of time preference such that $\rho > 0$.

Maximizing equation (3.14) subject to the agent's lifetime budget constraint yields the demand functions:

$$(3.15a) \quad a = \frac{\theta(1+\rho)}{(2+\rho)} w$$

$$(3.15b) \quad a_1 = \frac{\theta(1+r_a)}{(2+\rho)} w$$

$$(3.15c) \quad n = \frac{(1-\theta)(1+\rho)}{(2+\rho)} w$$

$$(3.15d) \quad n_1 = \frac{(1-\theta)(1+r_a)}{(2+\rho)} w$$

$$\text{where: } w = a^s + pn^s + T + \frac{a_1^s}{1+r_a} + \frac{pn_1^s}{1+r_a} + \frac{T_1}{1+r_a}$$

$$(3.16a) \quad T = (1 - \frac{P}{P^*}) q$$

$$(3.16b) \quad T_1 = (1 - \frac{P_1}{P_1^*}) q_1$$

In addition, this general equilibrium model also has the following two market clearing conditions; the demand for agricultural good in each period is equal to the supply of agricultural good in that period plus the agricultural import allowed by the quota in that period.

$$(3.17a) \quad a_0 = a_0^s + q_0$$

$$(3.17b) \quad a_1 = a_1^s + q_1$$

From the definition of the real interest rates described in chapter 2:

$$(2.6a) \quad (1 + r_a) \equiv (1 + i) \left(\frac{P_a}{P_{a1}} \right)$$

$$(2.6b) \quad (1 + r_n) \equiv (1 + i) \left(\frac{P_n}{P_{n1}} \right)$$

These two identities could be reduced to obtain

$$(3.18) \quad \frac{1+r_a}{1+r_n} = \frac{P_1}{P}$$

Note that we have a system of 8 equations $\{(2.29a), (3.15a) - (3.15d), (3.17a) - (3.17b), \text{ and } (3.18)\}$ and 8 endogenous variables $(a, a_1, n, n_1, tb, p, p_1, \text{ and } r_a)$.

To do the comparative static analysis of this general equilibrium model, we substitute equation (3.15a) into (3.17a) and obtain:

$$(3.19) \quad a^s + q_0 = \theta \left(\frac{1+\rho}{2+\rho} \right) \left[a^s + pn^s + \frac{a_1^s}{1+r_a} + \frac{pn_1^s}{1+r_n} + T_0 + \frac{T_1}{1+r_a} \right]$$

By using equations (2.4) and (3.16), equation (3.19) becomes

$$(3.20) \quad a^s + q_0 = \theta \left(\frac{1+\rho}{2+\rho} \right) \left[y + \frac{Y_1}{1+r_a} + \left(1 - \frac{p}{p^*}\right) q_0 + \frac{1}{1+r_a} \left(1 - \frac{p_1}{p_1^*}\right) q_1 \right]$$

With total differentiation of equation (3.20), we obtain

$$(3.21) \quad Af^a dp + f^a dA = \theta \left(\frac{1+\rho}{2+\rho} \right) [n^s dp + f^a dA + pf^n dN + \frac{n_1^s}{1+r_a} dp_1 \\ + \frac{1}{1+r_a} (f^a dA_1 + p_1 f^n dN_1) - \frac{Y_1}{(1+r_a)^2} dr_a - \frac{q_0}{p^*} dp \\ + \frac{p}{p^*} q_0 dp^* - \frac{q_1}{(1+r_a)^2} dr_a - \frac{q_1}{p_1^* (1+r_a)} dp_1 \\ + \frac{p_1 q_1}{p_1^* (1+r_a)^2} (p_1^* dr_a + (1+r_a) dp_1^*)]$$

Substituting equation (3.15) into (3.17a) and (3.17b),

$$a^s + q_0 = \theta \left(\frac{1+\rho}{2+\rho} \right) w$$

$$a_1^s + q_1 = \theta \left(\frac{1+r_a}{2+\rho} \right) w$$

These two equations could be reduced to

$$(3.22) \quad (a^s + q_0)(1+r_a) = (a_1^s + q_1)(1+\rho)$$

Total differentiating equation (3.22),

$$(3.23) \quad (1+r_a)(A f^{a'} dp + f^a dA) + (a^s + q_0) dr_a = (1+\rho)(A_1 f^{a'} dp_1 + f^a dA_1)$$

Total differentiating equation (3.18) and rewrite it to obtain

$$(3.24) \quad (1+r_a) dp - (1+r_n) dp_1 + p dr_a = p_1 dr_n$$

Equations (3.21), (3.23), and (3.24) form a system of three equations with three endogenous variables (p , p_1 , and r_a). Note that r_n is exogenous in this system. Since from equation (2.14) and (2.19), we could obtain

$$(1+i) = (1+i^*) \left(\frac{P_{n1} / P_{n1}^*}{P_{n0} / P_{n0}^*} \right)$$

or

$$(1+i) \frac{P_{n0}}{P_{n1}} = (1+i^*) \frac{P_{n0}^*}{P_{n1}^*}$$

And by using the definitions of the real interest rates shown in equations (2.20a), the above equation becomes

$$1+r_n = 1+r_n^*$$

Therefore, r_n is exogenous to the system.

Similar to the tariff-regime case, these equations will be evaluated at $g = g_1 = 0$ or $p = p^*$ and $p_1 = p_1^*$ when we conduct the comparative static analysis.

3.4 Effects of Disturbances

3.4.1 Productivity Changes

Similar to the tariff case, productivity shocks can be represented by appropriate changes in A_0 , A_1 , N_0 , and/or N_1 . However, the main difference is productivity shocks are transmitted across sectors and across time through both the actions of consumers on the demand side and producers on the supply side. The reason is domestic relative prices in both periods are no longer exogenous variables as in the tariff case. Therefore, any productivity shocks are transmitted across sectors and across time through prices, real interest rate, and the discounted value of the economic agent's real income stream. Furthermore, different sectors in which the shock occurs have different consequences on the trade balance.

3.4.1.1 Non-agricultural Sector's Productivity Changes

Similar to the tariff case, a positive non-agricultural sector productivity shock in either period (an increase in N_0 or N_1)

will have positive impacts on all demands under the quota regime as shown in Tables 3.1 - 3.2. The direct implication is that changes in current period demands could be observed without any change in current period income as the economy expects an improvement in her future non-agricultural sector productivity. There is also an important point that permanent shocks have larger consumption effects than temporary shocks.

On the supply side, unlike the tariff case, productivity shocks are transmitted across sectors and across time through relative prices. As shown in Tables 3.3 - 3.4, a current positive non-agricultural sector productivity shock increases the current agricultural and non-agricultural supplies. On the other hand, a future positive non-agricultural sector productivity shock will have a positive and negative supply-side effect on the current agricultural and non-agricultural sector, respectively.

The effect of a current non-agricultural productivity shock on the trade balance is ambiguous in general. Since the nation exports good n and imposes a quota on importing good a , the balance of trade measured in terms of the world price of good a (equations (2.29a) and (2.29b)) can be shown as:

$$(3.29a) \quad tb = p^* (n^j - n) - q_0$$

$$(3.29b) \quad tb_1 = p_1^* (n_1^j - n_1) - q_1$$

Formally differentiating equations (3.29) with respect to the

Table 3.1. Consumption effects of productivity shocks on current agricultural goods under a quota regime

$$\frac{da}{dA} = Af^{a'} \left(-\frac{1}{\Delta} f^a \left(-\frac{p}{p_1} a_1^s + \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) ((1+r_n)(a^s+q) - (1+\rho)A_1 f^{a'} p) \right) \right) + f^a > 0$$

$$\frac{da}{dA_1} = Af^{a'} \frac{1}{\Delta} \frac{1}{(1+r_a)} f^a \left((1+\rho) \frac{p}{p_1} a_1^s + (1+r_n)(a^s+q) - (1+\rho)pA_1 f^{a'} \right) > 0$$

$$\frac{da}{dA} + \frac{da}{dA_1} = f^a + Af^{a'} \left(f^a \frac{1}{\Delta} \left(((1+r_n)a - (1+\rho)pA_1 f^{a'}) \frac{(1+\theta + (1-\theta)(1+r_a))}{\theta(1+r_a)} \right) \right) > 0$$

$$\frac{da}{dN} = \frac{1}{\Delta} p(1+r_n) Af^{a'} f^n \left(a - \frac{(1+\rho)}{(1+r_n)} pA_1 f^{a'} \right) > 0$$

$$\frac{da}{dN_1} = \frac{1}{\Delta} \frac{p_1}{(1+r_a)} Af^{a'} f^n ((1+r_n)a - (1+\rho)pA_1 f^{a'}) > 0$$

$$\frac{da}{dN} + \frac{da}{dN_1} = \frac{1}{\Delta} f^n Af^{a'} ((1+r_n)a - (1+\rho)pA_1 f^{a'}) \left(p + \frac{p_1}{(1+r_a)} \right) > 0$$

$$\text{where: } \Delta = -a \left[n_1^s + n^s(1+r_n) - \frac{(2+\rho)(1+r_n)}{\theta(1+\rho)} Af^{a'} \right] + pf^{a'} \left(An_1^s + (1+\rho)A_1 n^s - \frac{(2+\rho)}{\theta} AA_1 f^{a'} \right) + \frac{Y_1 f^{a'}}{(1+r_a)} ((1+\rho)A_1 - (1+r_n)A) < 0$$

Table 3.2. Consumption effects of productivity shocks on current non-agricultural goods under a quota regime

$$\frac{dn}{dA} = -\frac{1}{\Delta} \frac{(1-\theta)}{\theta} f^a a (n_1^s + (1+r_n) n^s - (1+r_n) A f^{a'}) - (1+\rho) A_1 f^{a'} \left[n^s + \frac{Y_1}{(1+r_a)} \right] + (1+\rho) A A_1 p f^{a'^2} > 0$$

$$\frac{dn}{dA_1} = \frac{1}{\Delta} \frac{(1+\rho)(1-\theta)}{(2+\rho)(1+r_a)} f^a \left[-\left(\frac{(2+\rho)}{\theta} - 1 \right) \frac{p}{p_1} A f^{a'} a_1^s + a (n_1^s + n^s (1+r_n)) - (1+\rho) A_1 f^{a'} \left(p n^s + \frac{Y_1}{(1+r_a)} \right) \right] < 0$$

$$\begin{aligned} \frac{dn}{dA} + \frac{dn}{dA_1} &= \frac{1}{\Delta} (1-\theta) f^a \left[\left(\frac{\theta(1+\rho) - (2+\rho)(1+r_a)}{\theta(2+\rho)(1+r_a)} \right) a (n_1^s + (1+r_n) n^s) \right. \\ &\quad - \left(\frac{\theta(1+\rho) - (2+\rho)(1+r_a)}{\theta(2+\rho)(1+r_a)} \right) A_1 f^{a'} \left[n^s p + \frac{Y_1}{(1+r_a)} \right] \\ &\quad - A f^{a'} \left[\frac{(1+\rho)}{\theta} p A_1 f^{a'} + \frac{(1+\rho)}{(1+r_a)} \left(\frac{1}{\theta} - \frac{1}{(2+\rho)} \right) \frac{p}{p_1} a_1^s \right. \\ &\quad \left. \left. - \frac{(1+r_n)}{\theta} a \right] \right] > 0 \end{aligned}$$

$$\frac{dn}{dN} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} A f^{a'} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) > 0$$

$$\frac{dn}{dN_1} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta(1+r_a)} p_1 f^n A f^{a'} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) > 0$$

$$\frac{dn}{dN} + \frac{dn}{dN_1} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} A f^{a'} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) \left(1 + \frac{p_1}{(1+r_a)} f^n \right) > 0$$

where: $\Delta = -a \left[n_1^s + n^s (1+r_n) - \frac{(2+\rho)(1+r_n)}{\theta(1+\rho)} A f^{a'} \right]$
 $+ p f^{a'} \left[A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right]$
 $+ \frac{Y_1 f^{a'}}{(1+r_a)} ((1+\rho) A_1 - (1+r_n) A) < 0$

Table 3.3. Production effects of productivity shocks on current agricultural goods under a quota regime

$$\frac{da^s}{dA} = f^a + Af^{a'} - \frac{1}{\Delta} f^a \left(-\frac{P}{P_1} a_1^s + \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) ((1+r_n)(a^s+q) - (1+\rho)pA_1f^{a'}) \right) > 0$$

$$\frac{da^s}{dA_1} = Af^{a'} \frac{1}{\Delta} \frac{1}{(1+r_a)} f^a \left((1+\rho) \frac{P}{P_1} a_1^s + (1+r_n)(a^s+q) - (1+\rho)pA_1f^{a'} \right) > 0$$

$$\begin{aligned} \frac{da^s}{dA} + \frac{da^s}{dA_1} &= f^a + \frac{1}{\Delta} f^a Af^{a'} \left[\frac{P}{P_1} a_1^s \left(\frac{1+\rho}{1+r_a} + 1 \right) \right. \\ &\quad \left. + ((1+r_n)a - (1+\rho)pA_1f^{a'}) \left(-1 + \frac{1}{1+r_a} + \frac{2+\rho}{(1+\rho)\theta} \right) \right] > 0 \end{aligned}$$

$$\frac{da^s}{dN} = Af^{a'} \frac{1}{\Delta} p(1+r_n) f^n (a - pA_1f^{a'}) > 0$$

$$\frac{da^s}{dN} + \frac{da^s}{dN_1} = \frac{1}{\Delta} Pf^n Af^{a'} (2+r_n) \left(a - \frac{1+\rho}{1+r_n} pA_1f^{a'} \right) > 0$$

$$\frac{da^s}{dN_1} = \frac{1}{\Delta} \frac{P_1}{(1+r_a)} f^n Af^{a'} (1+r_n) (a - (1+\rho)pA_1f^{a'}) > 0$$

where:
$$\Delta = -a \left[n_1^s + n^s(1+r_n) - \frac{(2+\rho)(1+r_n)}{\theta(1+\rho)} Af^{a'} \right] + pf^{a'} \left[An_1^s + (1+\rho)A_1n^s - \frac{(2+\rho)}{\theta} AA_1f^{a'} \right] + \frac{Y_1f^{a'}}{(1+r_a)} ((1+\rho)A_1 - (1+r_n)A) < 0$$

Table 3.4. Production effects of productivity shocks on
current non-agricultural goods
under a quota regime

$$\frac{dn^s}{dA} = Nf^{n'} - \frac{1}{\Delta} f^a \left(-\frac{P}{P_1} a_1^s + (1 - \frac{(2+\rho)}{(1+\rho)\theta}) ((1+r_n)(a^s+q) - (1+\rho)pA_1 f^{a'}) \right) < 0$$

$$\frac{dn^s}{dA_1} = Nf^{n'} \frac{1}{\Delta} \frac{1}{(1+r_a)} f^a \left((1+\rho) \frac{P}{P_1} a_1^s + (1+r_n)(a^s+q) - (1+\rho)pA_1 f^{a'} \right) < 0$$

$$\begin{aligned} \frac{dn^s}{dA} + \frac{dn^s}{dA_1} &= \frac{1}{\Delta} f^a Nf^{n'} \left[\frac{P}{P_1} a_1^s \left(\frac{1+\rho}{1+r_a} + 1 \right) \right. \\ &\quad \left. + ((1+r_n)a - (1+\rho)pA_1 f^{a'}) \left(-1 + \frac{1}{1+r_a} + \frac{2+\rho}{(1+\rho)\theta} \right) \right] < 0 \end{aligned}$$

$$\frac{dn^s}{dN} = Nf^{n'} \frac{1}{\Delta} p(1+r_n) f^n (a - pA_1 f^{a'}) < 0$$

$$\frac{dn^s}{dN_1} = Nf^{n'} \frac{1}{\Delta} \frac{P_1}{(1+r_a)} f^n ((1+r_n)(a - (1+\rho)pA_1 f^{a'})) < 0$$

$$\frac{dn^s}{dN} + \frac{dn^s}{dN_1} = f^n + \frac{1}{\Delta} P f^n Nf^{n'} (2+r_n) \left(a - \frac{1+\rho}{1+r_n} pA_1 f^{a'} \right) < 0$$

where: $\Delta = -a \left[n_1^s + n^s(1+r_n) - \frac{(2+\rho)(1+r_n)}{\theta(1+\rho)} A f^{a'} \right]$
 $+ p f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)$
 $+ \frac{Y_1 f^{a'}}{(1+r_a)} ((1+\rho) A_1 - (1+r_n) A) < 0$

Table 3.5. Consumption effects of productivity shocks on current trade balance under a quota regime

$$\begin{aligned} \frac{dtb}{dA} = & \frac{1}{\Delta} p^* f^a \left[N f^n \left(\frac{p}{p_1} a_1^s - \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) ((1+r_n) a - (1+\rho) p A_1 f^{a'}) \right) \right. \\ & + \frac{(1-\theta)}{\theta} [a (n_1^s + (1+r_n) n^s - (1+r_n) A f^{a'}) \\ & \left. - (1+\rho) f^a A_1 f^{a'} (p n^s + \frac{y_1}{(1+r_a)}) + (1+\rho) p A A_1 f^{a/2}] \right] < 0 \end{aligned}$$

$$\begin{aligned} \frac{dtb}{dA_1} = & \frac{1}{\Delta} \frac{p^* f^a}{(1+r_a)} \left[\frac{p}{p_1} a_1^s \left((1+\rho) N f^{n'} + \left(\frac{(2+\rho)}{\theta} - 1 \right) A f^{a'} \right) \right. \\ & + a \left((1+r_n) N f^{n'} - (n_1^s + (1+r_n) n^s) \right) \\ & \left. + (1+\rho) A_1 f^a \left(-p N f^{n'} + \left(n^s p + \frac{y_1}{(1+r_a)} \right) \right) \right] = ? \end{aligned}$$

$$\begin{aligned} \frac{dtb}{dA} + \frac{dtb}{dA_1} = & \frac{1}{\Delta} p^* f^a \left[N f^n \left(\frac{(1+r_n)}{(1+r_a)} a + \left(1 + \frac{(1+\rho)}{(1+r_a)} \right) \frac{p}{p_1} a_1^s \right) \right. \\ & - \frac{(1-\theta)}{\theta} (1+r_n) A f^{a'} a + (1+\rho) p A_1 f^a \left(-\frac{1}{(1+r_a)} N f^{n'} + \frac{(1-\theta)}{\theta} A f^{a'} \right) \\ & + (1+\rho) A_1 f^a \left(p n^s + \frac{y_1}{(1+r_a)} \right) \left(\frac{1}{(1+r_a)} - \frac{(1-\theta)}{\theta} f^a \right) \\ & \left. + a (n_1^s + (1+r_n) n^s) \left(-\frac{1}{(1+r_a)} + \frac{(1-\theta)}{\theta} \right) \right] = ? \end{aligned}$$

$$\frac{dtb}{dN} = p^* \left(f^n + \frac{1}{\Delta} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) (N f^{n'} f^n p - \frac{(1-\theta)}{\theta} A f^{a'}) \right) = ?$$

$$\frac{dtb}{dN_1} = \frac{1}{\Delta} p^* \frac{p_1}{(1+r_a)} f^n ((1+r_n) a - (1+\rho) p A_1 f^{a'}) \left(N f^{n'} - \frac{(1-\theta)}{\theta} A f^{a'} \right) < 0$$

$$\begin{aligned} \frac{dtb}{dN} + \frac{dtb}{dN_1} = & p^* f^n + \frac{1}{\Delta} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) \left[p^* f^n N f^n \left(p + \frac{p_1}{(1+r_a)} \right) \right. \\ & \left. - \frac{(1-\theta)}{\theta} p^* A f^{a'} \left(1 + \frac{p_1}{(1+r_a)} f^n \right) \right] = ? \end{aligned}$$

non-agricultural shock:

$$\begin{aligned}
 (3.30a) \quad \frac{d(tb)}{dN} &= p \cdot \left(f^n + Nf^{n'} \frac{dp}{dN} - \frac{dn}{dN} \right) \\
 &= p \cdot \left(f^n + \frac{1}{\Delta} \left((1+r_n) a_0 - (1+\rho) p A_1 f^{a'} \right) (Nf^{n'} p f^n - \frac{(1-\theta)}{\theta} A f^{a'}) \right) \\
 &= ?
 \end{aligned}$$

$$\begin{aligned}
 (3.30b) \quad \frac{d(tb)}{dN_1} &= p \cdot \left[Nf^{n'} \frac{dp}{dN_1} - \frac{dn}{dN_1} \right] \\
 &= \frac{1}{\Delta} p \cdot \frac{p_1}{(1+r_a)} f^n \left((1+r_n) a_0 - (1+\rho) p A_1 f^{a'} \right) \left(Nf^{n'} - \frac{(1-\theta)}{\theta} A f^{a'} \right) \\
 &< 0
 \end{aligned}$$

In general, the sign of equation (3.30a) is ambiguous. For a nation exporting good n , a current positive non-agricultural shock directly increases the nation's exportable non-agricultural supply, and so improves the trade balance. On the other hand, it causes the relative price of good n to decline, which has negative supply-side and positive demand-side effects on good n . In addition, the increased wealth also causes the agent to consume more of both goods, and so worsens the trade balance.

Furthermore, similar to the tariff case, a positive non-agricultural sector productivity shock in the future period will worsen the current trade balance. As shown in equation (3.30b), the increased productivity causes the current relative price of good n to decline, which has negative supply-side and positive demand-side effects on the exportable good n . In addition, the increased future income causes the agent to consume more of both goods, and so further worsens the trade balance.

By differentiating equation (3.29a) with respect to the permanent non-agricultural shock:

$$(3.31) \quad \frac{d(tb)}{dN} + \frac{d(tb)}{dN_1} = p^* \left[f^n + Nf^{n'} \left(\frac{dp}{dN} + \frac{dp}{dN_1} \right) - \left(\frac{dn}{dN} + \frac{dn}{dN_1} \right) \right]$$

For the specific utility function, substitute the entries listed in Appendix B into equation (3.31) to obtain:

$$\begin{aligned} \frac{d(tb)}{dN} + \frac{d(tb)}{dN_1} &= p^* f^n + \frac{1}{\Delta} \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \left[p^* f^n N f^{n'} \left(p + \frac{p_1}{(1+r_a)} \right) \right. \\ &\quad \left. - \frac{(1-\theta)}{\theta} p^* A f^a \left(1 + \frac{p_1}{(1+r_a)} f^n \right) \right] = ? \end{aligned}$$

Therefore, permanent non-agricultural sector productivity changes have an ambiguous effect on the trade balance; if income levels in both periods increase, there is no presumption as to whether economic agents will save or dissave.

3.4.1.2 Agricultural Sector's Productivity Changes

Similar to the tariff case, a positive agricultural sector productivity shock in the current period (an increase in A_0) will have positive impacts on all demands. As shown in Tables 3.1 - 3.2, however, a future positive agricultural productivity change will decrease the demands for non-agricultural goods in both periods, but increase the current demand for agricultural goods and ambiguously affect the future demand for agricultural goods. Differentiating the domestic non-agricultural and agricultural demand equations in each period with respect to A_1 :

$$(3.32a) \quad \frac{dn}{dA_1} = \frac{1}{\Delta} \frac{(1+\rho)(1-\theta)}{(2+\rho)(1+r_a)} f^a \left[- \left(\frac{(2+\rho)}{\theta} - 1 \right) \frac{p}{p_1} A f^{a'} a_1^s + a(n_1^s + n^s(1+r_n)) - (1+\rho) A_1 f^{a'} \left(p n^s + \frac{y_1}{(1+r_a)} \right) \right] < 0$$

$$(3.32b) \quad \frac{dn_1}{dA_1} = \frac{1}{\Delta} \frac{(1-\theta)}{(2+\rho)} f^a \left[- \left(\frac{(2+\rho)}{\theta} - 1 \right) \frac{p}{p_1} A f^{a'} a_1^s + a(n_1^s + n^s(1+r_n)) - (1+\rho) A_1 f^{a'} \left(p n^s + \frac{y_1}{(1+r_a)} \right) \right] < 0$$

$$(3.32c) \quad \frac{da}{dA_1} = \frac{1}{\Delta} \frac{1}{(1+r_a)} A f^{a'} f^a \left((1+\rho) \frac{p}{p_1} a_1^s + (1+r_n) a - (1+\rho) p A_1 f^{a'} \right) > 0$$

$$(3.32d) \quad \frac{da_1}{dA_1} = \frac{1}{\Delta} A_1 f^{a'} f^a \left[- (1+\rho) \left(y + \frac{y_1}{(1+r_a)} \right) + \left(\frac{(2+\rho)}{\theta} - 1 \right) p A f^{a'} + a + (1+\rho) a^s \right] + f^a = ?$$

On the supply side, unlike the tariff case, productivity shocks are transmitted across sectors and across time through relative prices. As shown in Tables 3.3 - 3.4, a current positive agricultural-sector productivity shock increases and decreases the current agricultural and non-agricultural supplies, respectively. In addition, a future positive agricultural sector productivity shock will have a positive and negative supply-side effects on the current agricultural and non-agricultural sector, respectively.

The effect of a current agricultural productivity shock on the trade balance is unambiguously negative as shown in Table 3.5. However, its future shock has an ambiguous impact on the trade balance. To illustrate these results, formally differentiating equations (3.29a) with respect to the current and future agricultural shocks:

$$\begin{aligned}
 (3.33a) \quad \frac{d(tb)}{dA} &= p^* \left(Nf^{n'} \frac{dp}{dA} - \frac{dn}{dA} \right) \\
 &= \frac{1}{\Delta} p^* f^a \left[Nf^n \left(\frac{p}{p_1} a_1^s - \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) ((1+r_n) a - (1+\rho) p A_1 f^{a'}) \right) \right. \\
 &\quad + \frac{(1-\theta)}{\theta} [a(n_1^s + (1+r_n) n^s - (1+r_n) A f^{a'}) \\
 &\quad \left. - (1+\rho) f^a A_1 f^{a'} (p n^s + \frac{y_1}{(1+r_a)}) + (1+\rho) p A A_1 f^{a/2}] \right] < 0
 \end{aligned}$$

$$(3.33b) \quad \frac{d(tb)}{dA_1} = p^* \left(Nf^{n'} \frac{dp}{dA_1} - \frac{dn}{dA_1} \right)$$

$$\begin{aligned}
&= \frac{1}{\Delta} \frac{p^* f^a}{(1+r_a)} \left[\frac{p}{p_1} a_1^s \left((1+\rho) N f^{n'} + \left(\frac{(2+\rho)}{\theta} - 1 \right) A f^a \right) \right. \\
&\quad + a \left((1+r_n) N f^{n'} - (n_1^s + (1+r_n) n^s) \right) \\
&\quad \left. + (1+\rho) A_1 f^a \left(-p N f^{n'} + (n^s p + \frac{y_1}{(1+r_a)}) \right) \right] = ?
\end{aligned}$$

As shown in equation (3.33a), the increased productivity causes the current relative price of good n to decline, which has negative supply-side and positive demand-side effects on the exportable good n . In addition, the increased income and the declining real interest rate cause the agent to consume more of both goods, and so further worsens the trade balance.

In general, the sign of equation (3.33b) is ambiguous. For a nation exporting good n , a future positive agricultural shock causes the relative price of good n to decline, which has negative supply-side and positive demand-side effects on good n . In addition, the increased wealth also causes the agent to consume more of both goods, and so worsens the trade balance. On the other hand, the rising real interest rate induces the economic agent to save, and hence reduces demands on both goods. For this specific model, the net effect on the domestic demand for non-agricultural goods is negative. Therefore, the impact on the trade balance is ambiguous.

Furthermore, permanent agricultural sector productivity changes have an ambiguous effect on the trade balance; if income levels in both periods increase, there is no presumption as to whether economic agents will save or

dissave. By differentiating equation (3.29a) with respect to the permanent agricultural shock:

$$\begin{aligned}
 (3.34) \quad \frac{d(tb)}{dA} + \frac{d(tb)}{dA_1} &= p^* \left[Nf^{n'} \left(\frac{dp}{dA} + \frac{dp}{dA_1} \right) - \left(\frac{dn}{dA} + \frac{dn}{dA_1} \right) \right] \\
 &= \frac{1}{\Delta} p^* f^a \left[Nf^{n'} \left(\frac{(1+r_n)}{(1+r_a)} a + \left(1 + \frac{(1+p)}{(1+r_a)} \right) \frac{p}{p_1} a_1^s \right) \right. \\
 &\quad - \frac{(1-\theta)}{\theta} (1+r_n) Af^{a'} a + (1+p) p A_1 f^{a'} \left(-\frac{1}{(1+r_a)} Nf^{n'} + \frac{(1-\theta)}{\theta} Af^{a'} \right) \\
 &\quad + (1+p) A_1 f^{a'} \left(p n^s + \frac{y_1}{(1+r_a)} \right) \left(\frac{1}{(1+r_a)} - \frac{(1-\theta)}{\theta} f^a \right) \\
 &\quad \left. + a (n_1^s + (1+r_n) n^s) \left(-\frac{1}{(1+r_a)} + \frac{(1-\theta)}{\theta} \right) \right] = ?
 \end{aligned}$$

Therefore, permanent non-agricultural sector productivity changes have an ambiguous effect on the trade balance.

The important point is that, similar to the tariff case, there is no simple relationship between productivity and the trade balance. Permanent productivity changes in either the import or the export sectors may generate trade surpluses or deficits. Moreover, current period productivity in the import sector may be high but the trade balance may exhibit a deficit.

3.4.2 External Disturbances

The small open economy takes p^* , r_a^* , and r_n^* as given.

However, for a given quota schedule, relative prices can change through disturbances on both domestic and foreign markets.

3.4.2.1 Permanent Changes in the Terms of Trade As

shown in Tables 3.6 - 3.7, a permanent increase in the nation's terms of trade will increase the supply of good n and decrease the supply of good a in the current period. For the specific log-linear utility function, the effects of a change in relative prices are listed in the first row of Tables 3.6 - 3.9 where, for comparability with the tariff case, all total derivatives are evaluated at the point $T_0 = T_1 = 0$. As seen in the table, a permanent increase in the relative price of good n reduces all demands in each period.

It is crucial to note that, unlike the tariff case, an improvement in a nation's terms of trade will improve a nation's trade balance. The reason is a reduction in the demand for good n coupled with an increase in supply results in an overall increase in the export supply. In addition, the demand for the importable good a is forced to decline by the declining domestic supply and the unchanged level of import quota, which results in an unchange in import demand.

From equation (3.29a):

$$(3.29a) \quad tb = p^* (n^s - n) - q_0$$

Table 3.6. Effects of relative price changes on current agricultural goods under a quota regime

$$\frac{da}{dp^*} = Af^{a'} < 0$$

$$\frac{da}{dr_n^*} = \frac{1}{\Delta} \frac{P_1}{(1+r_a)} Af^{a'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} y_1 A_1 f^{a'} \right) < 0$$

$$\frac{da}{dr_a^*} = \frac{\frac{1}{(1+r_a)} Af^{a'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} < 0$$

$$\frac{da^s}{dp^*} = Af^{a'} < 0$$

$$\frac{da^s}{dr_n^*} = \frac{1}{\Delta} \frac{P_1}{(1+r_a)} Af^{a'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} y_1 A_1 f^{a'} \right) < 0$$

$$\frac{da^s}{dr_a^*} = \frac{\frac{1}{(1+r_a)} Af^{a'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} < 0$$

Table 3.7. Effects of relative price changes on current non-agricultural goods under a quota regime

$$\frac{dn}{dp^*} = \frac{(1-\theta)}{\theta} Af^{a'} < 0$$

$$\frac{dn}{dr_n^*} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta(1+r_a)} p_1 Af^{a'} (-n_1^s a + (1+\rho) A_1 f^{a'} \frac{y_1}{(1+r_a)}) < 0$$

$$\frac{dn}{dr_a^*} = \frac{\frac{(1-\theta)}{\theta(1+r_a)} A \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{An_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} AA_1 f^{a'}} < 0$$

$$\frac{dn^s}{dp^*} = Nf^{n'} > 0$$

$$\frac{dn}{dr_n^*} = \frac{1}{\Delta} \frac{p_1}{(1+r_a)} Nf^{n'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} y_1 A_1 f^{a'} \right) > 0$$

$$\frac{dn^s}{dr_a^*} = \frac{\frac{1}{(1+r_a)} Nf^{n'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{f^{a'} \left(An_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} AA_1 f^{a'} \right)} > 0$$

Table 3.8. Effects of relative price changes on future agricultural goods under a quota regime

$$\frac{da_1}{dp^*} = \frac{(1+r_a) A_1 f^a \left(a \left(n^s - \frac{(2+\rho)}{(1+\rho)\theta} A f^a \right) + \frac{A f^{a'} y_1}{(1+r_a)} \right)}{\left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)} < 0$$

$$\frac{da_1}{dr_n^*} = \frac{1}{\Delta} p_1 A_1 f^{a'} \left(\frac{(1+\rho)\theta}{(2+\rho)} n^s w - y A f^a \right) > 0$$

$$\frac{da_1}{dr_a^*} = \frac{A_1 f^{a'} \left(n^s a - \frac{(2+\rho)}{(1+\rho)\theta} A f^{a'} a + \frac{A f^{a'}}{(1+r_a)} \right)}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} > 0$$

Table 3.9. Effects of relative price changes on future non-agricultural goods under a quota regime

$$\frac{dn_1}{dp^*} = \frac{(1-\theta)}{(1+\rho)\theta} (1+r_a) A f^{a'} < 0$$

$$\frac{dn_1}{dr_n^*} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} p_1 A f^a \left(-\frac{n_1^s a}{(1+\rho)} + A_1 f^{a'} \frac{y_1}{(1+r_a)} \right) < 0$$

$$\frac{dn_1}{dr_a^*} = \frac{\frac{(1-\theta)}{\theta(1+\rho)} A \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'}} < 0$$

Table 3.10. Effects of relative price changes on current trade balance under a quota regime

$$\frac{dtb}{dp^*} = (n^s - n) - p^* \left(\frac{(1-\theta)}{\theta} A f^{a'} - N f^{n'} \right) > 0$$

$$\frac{dtb}{dr_n^*} = \frac{1}{\Delta} p_1 \left(\frac{p^*}{(1+r_a)} n_1^s a - A_1 f^{a'} \frac{(1+\rho)}{(1+r_a)^2} p^* y_1 \right) \left(\frac{(1-\theta)}{\theta} A f^{a'} - N f^{n'} \right) > 0$$

$$\frac{dtb}{dr_a^*} = \frac{(1-\theta)}{(1+r_a)} \frac{p^* \left(n_1^s a - \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right) \left(\frac{A f^{a'}}{\theta} - \frac{N f^{n'}}{(1-\theta)} \right)}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} > 0$$

Formally differentiating equation (3.29a) with respect to the terms of trade:

$$\begin{aligned}
 (3.35) \quad \frac{d(tb)}{dp^*} &= (n^s - n) + p^* \left[Nf^{n'} \frac{dp}{dp^*} - \frac{dn}{dp^*} \right] \\
 &= (n^s - n) - p^* \left(\frac{(1-\theta)}{\theta} Af^{a'} - Nf^{n'} \right) > 0
 \end{aligned}$$

The trade balance is the difference between total domestic production ($a' + p'n'$) and total domestic expenditures ($a + p'n$). The intuitive explanation of the positive effect of a permanent improvement in the terms of trade on the trade balance is that there is an incentive for agents to intertemporally relocate expenditures in response to a permanent change in relative prices. The key reason is, under a quota regime, the government effectively limits the nation's borrowing (dissaving) from the international market by predetermining the quota level.

3.4.2.2 Interest Rate Changes The increase in real interest rates causes current relative prices to rise, which increases and decreases the supplies of exportable and importable goods, respectively. It also induces a substitution away from present consumption towards future consumption. In this circumstance, the trade balance improves since quota is unchanged and current period expenditures on good a and n decline.

3.4.2.3 Temporary Price Changes

As discussed in Chapter 2, temporary changes in commodity prices can be viewed as a combination of a real interest rate change and a change in the terms of trade. A temporary increase in the relative price of good n (i.e., an increase in p^* without any change in p_1^*) does not involve a change in $(1+i^*)$ or $(1+r_n^*)$ but r_n^* changes such that $p^*/(1+r_n^*)$ remains constant. Current period production of the exportable good n increases at the expense of the importable good a . Furthermore, demand for good n declines and the demand for good a is forced to decrease by the declining domestic supply and the unchanged level of import quota. The net impact is the trade balance improves. The second and third rows of Tables 3.6 - 3.9 give the effects of pure changes in interest rates on commodity demands for the specific log-linear utility function.

3.5 Money and Exchange Rate Neutrality

A similar implication of this study to the tariff case is that it exhibits exchange rate/money neutrality. Notice that the behavior of the real variables in the system could be determined without reference to the nominal exchange rate or to the money supply. On the other hand, prices and the exchange rate are determined by movements in the real variables in the system.

Using the same method as the tariff case, we solve for

the nominal variables by substituting the domestic demand for domestic money (equation (2.8)) and the foreign demand for domestic money (equation (2.17)) into the money market clearing condition (equation (2.18)). In the initial period, money market equilibrium entails:

$$(3.36) \quad M = p_a a^s + p_n n + \left[1 - \frac{p}{p^*}\right] q + p_n (n^s - n) \\ = e p_a^* (a^s + p^* n^s) + \left(1 - \frac{p}{p^*}\right) q$$

Since this general equilibrium model consists of a nonlinear-equation system. Instead of attempting to solve for the nominal variables directly, we will demonstrate the issue by utilizing the comparative static analyses shown in Appendix B.

Total differentiating equation (3.36), we obtain

$$(3.37) \quad dM = p_a^* (a^s + p^* n^s) de + e p_a^* f^a dA + p^* f^n dN \\ + e (a^s + p^* n^s) dp_a^* + \frac{p}{p^{*2}} q dp^* \\ + \left(e p_a^* (A f^{a'} + p^* N f^{n'}) - \frac{q}{p^*} \right) dp$$

where:

$$dp = \frac{1}{\Delta_1} \left[f^a \left(p \left(n_1^s - \frac{q_1}{p_1^*} \right) - \frac{(1+r_n)}{(1+r_a)} (T_1 + y_1) \right) \right. \\ \left. + \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) f^a \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \right] dA$$

$$\begin{aligned}
& + \frac{1}{\Delta_1} \left[\frac{(1+\rho)}{(1+r_a)} f^a \left(p \left(n_1^s - \frac{q_1}{p_1^*} \right) - \frac{(1+r_n)}{(1+r_a)} (T_1 + y_1) \right) \right. \\
& + \frac{f^a}{(1+r_a)} ((1+r_n) a - (1+\rho) p A_1 f^{a'})] dA_1 \\
& + \frac{1}{\Delta_1} p f^n ((1+r_n) a - (1+\rho) p A_1 f^{a'}) dN \\
& + \frac{1}{\Delta_1} \frac{1}{(1+r_a)} p_1 f^n ((1+r_n) a - (1+\rho) p A_1 f^{a'}) dN_1 \\
& + \frac{1}{\Delta_1} \frac{p_1}{(1+r_a)} \left(a \left(-n_1^s + \frac{q_1}{p_1^*} \right) + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} (T_1 + y_1) \right) dr_n^* \\
& + \frac{1}{\Delta_1} \frac{pq}{p^{*2}} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) dp^* \\
& + \frac{1}{\Delta_1} \frac{p_1 q_1}{(1+r_a) p_1^{*2}} ((1+r_n) a - (1+\rho) p A_1 f^{a'}) dp_1^*
\end{aligned}$$

$$\Delta_1 = a \left(-n_1^s + \frac{q_1}{p_1^*} - (1+r_n) n^s + \frac{(1+r_n)}{p^*} q \right)$$

$$\begin{aligned}
& + p f^a \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(1+\rho)}{p^*} A_1 q - \frac{A q_1}{p_1^*} - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right) \\
& + \frac{T_1 f^{a'}}{(1+r_a)} (A_1 (1+\rho) - A (1+r_n)) + \frac{y_1 f^{a'}}{(1+r_a)} (A_1 (1+\rho) - A (1+r_n)) \\
& + \frac{(2+\rho) (1+r_n)}{\theta (1+\rho)} A f^{a'} a
\end{aligned}$$

As shown above, equation (3.37) doesn't consist of any other endogenous variables (e.g., da , dn , da_1 , dn_1 , and $d(tb)$) except the nominal exchange rate. Therefore, the nominal exchange rate is directly proportional to the domestic money supply and indirectly proportional to p_a^* , p_{a1}^* , p_n^* , and p_{n1}^* . In addition, the exchange rate depends on current and future values of the real variables in the system. An increase in current output of either good will act to appreciate the domestic currency as increased transactions increase the demand for the domestic currency.

The crucial point is that the exchange rate is completely endogenous; co-movements between the nominal exchange rate and other variables in the system cannot be attributed to the exchange rate itself. Moreover, there is no simple relationship between the exchange rate and the other endogenous variables in the system. It cannot be concluded that the decline in the trade balance is due to movements in the nominal exchange rate.

IV. CONCLUSIONS

The main purpose of this dissertation is the development of an intertemporal-optimizing general equilibrium model of exchange rates and external imbalances by providing a bridge between recent developments in international finance literature and international trade and policy literature.

Using an intertemporal-optimizing general-equilibrium framework, many presumptions of the traditional international finance literature have been overturned; the current account should be viewed as a nation's decision to save or dissave, and therefore must be analyzed in an intertemporal context. Another virtue of macro-models derived from individual optimizing behavior is that the behavioral rules of economic agents are consistent with the performance of the macroeconomic model.

The model is one of a small-open economy producing an agricultural and a non-agricultural good in each of two periods. The nation is an importer of agricultural goods and the government uses trade policy to protect its agricultural sector. Production decisions are made so as to maximize profits. The model is general equilibrium in nature; households are the ultimate owners of the firms and view all production as income. Each household is forward-looking in the sense that it maximizes lifetime utility subject to an intertemporal lifetime budget constraint. All transactions

require the use of money so that it is possible to consider the co-movements of the nominal exchange rate and real economic variables. Within this framework, it is also possible to analyze the effects of various macroeconomic shocks on the household. Particular emphasis is placed on the role of tariffs and quotas.

We consider the optimization problems of households and firms. The general equilibrium model is solved. The effects of internal and external macroeconomic disturbances are then considered.

Another concern of this dissertation is the consideration of the equivalence of tariffs and quotas. The study shows that as long as the government captures the quota revenue, collects the revenue in domestic currency, and rebates the proceeds to the public in lump-sum fashion, there is a vector of quotas which is equivalent to any arbitrary tariff vector. Nevertheless, the equivalence breaks down when internal and external economic disturbances are considered.

Furthermore, the model exhibits exchange rate and money neutrality. It shows that exchange rate is completely endogenous; co-movements between the nominal exchange rate and other variables in the system cannot be attributed to the exchange rate itself. Moreover, there is no simple relationship between the exchange rate and the other endogenous variables in the system. In no sense can it be

said that the decline in the trade balance is due to movements in the nominal exchange rate.

The model has some other obvious applications and extensions. The small-country assumption could be relaxed by introducing a downward sloping foreign demand function for the domestic export. Furthermore, although the two-period model clearly shows the nation's obligation to repay her debt in the future period, it can also be extended to consider a multi-period model which could demonstrate the persistence of the external debt occurring in the real world. Introducing uncertainty would provide a more realistic description of actual economies and would provide an interesting vehicle for considering the world's debt problem. In the model presented here, all debt obligations were undertaken with perfect foresight. Another interesting aspect would be to add an element of price stickiness in the model.

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APPENDIX A: DEFINITION OF VARIABLES

Since there are only two time periods ($i = 0, 1$), as a notational convenience the subscript zero was dropped when it's unambiguous to do so.

a_i = consumption of the agricultural good (good a) in period i.

n_i = consumption of the non-agricultural good (good n) in period i.

a_i^s = production of good a in period i ($a_i^s = A_i f^a$).

n_i^s = production of good n in period i ($n_i^s = N_i f^n$).

f^n = $df^n/dp = df^n/dp_i$

f^a = $df^a/dp = df^a/dp_i$

p_{ai} = domestic nominal price of good a in period i.

p_{ai}^* = foreign nominal price of good a in period i.

p_{ni} = domestic nominal price of good n in period i.

p_{ni}^* = foreign nominal price of good n in period i.

p_i = domestic relative price of good n in period i
($p_i = p_{ni}/p_{ai}$).

p_i^* = foreign relative price of good n in period i
($p_i^* = p_{ni}^*/p_{ai}^*$).

A_i = multiplicative productivity term acting to shift the supply of good a in period i.

N_i = multiplicative productivity term acting to shift the supply of good n in period i.

- y_i = total production valued in terms of the domestic relative price of good a in period i.
- i = domestic nominal interest rate on borrowing (saving).
- i^* = foreign nominal interest rate.
- r_a = domestic real interest rate measured in terms of good a.
- r_a^* = foreign real interest rate measured in terms of good a.
- r_n = domestic real interest rate measured in terms of good n.
- r_n^* = foreign real interest rate measured in terms of good n.
- u = the economic agent's lifetime utility function from the consumption of each good in each period of life.
- θ = a share parameter; $0 < \theta < 1$.
- ρ = the rate of time preference.
- m = number of units of domestic currency demanded by domestics.
- e = domestic currency price of foreign exchange.
- t_i = ad valorem tariff rate on good a in period i.
- $t_i e_i p_a^*$ = tariff per unit of a imported in period i.
- m_i^f = demand for the foreign currency in period i.
- s = domestic saving.
- T_i = transfer received from the government in period i.
- m_i^{*f} = foreign demand for the domestic currency in period i.
- M_i = total available stock of nominal money in period i.
- w = discounted value of the economic agent's lifetime real income stream.

tb_i = balance of trade measured in terms of the world price of good a.

ϵ_x = total price elasticity of demand for good x.

g_i = measure of the price gap between the domestic and the foreign price that is introduced by the quota in period i ($1+g_i=p_{xi}/e_i p_{xi}^*$).

p_{qi} = competitively determined price of import licenses in period i.

q_i = import quota in period i.

APPENDIX B:

THE COMPARATIVE STATIC ANALYSIS OF THE SYSTEM

UNDER A QUOTA REGIME

The economic agent maximizes his/her log-linear intertemporal utility function:

$$(3.14) \quad u = \theta \ln a_0 + (1-\theta) \ln n_0 + \frac{\theta}{(1+\rho)} \ln a_1 + \frac{(1-\theta)}{(1+\rho)} \ln n_1$$

where: θ is a share parameter such that $0 < \theta < 1$; and ρ is the subjective rate of time preference such that $\rho > 0$.

Maximizing equation (3.14) subject to the agent's intertemporal budget constraint yields the demand functions:

$$(3.15a) \quad a = \frac{\theta(1+\rho)}{(2+\rho)} w$$

$$(3.15b) \quad a_1 = \frac{\theta(1+r_a)}{(2+\rho)} w$$

$$(3.15c) \quad n = \frac{(1-\theta)(1+\rho)}{(2+\rho)} w$$

$$(3.15d) \quad n_1 = \frac{(1-\theta)(1+r_a)}{(2+\rho)} w$$

$$\text{where: } w = a^s + pn^s + T + \frac{a_1^s}{1+r_a} + \frac{pn_1^s}{1+r_n} + \frac{T_1}{1+r_a}$$

Rewriting equations (3.11) by using the definition of the quota revenues as

$$(3.16a) \quad T \equiv (1 - \frac{P}{P^*})q$$

$$(3.16b) \quad T_1 \equiv (1 - \frac{P_1}{P_1^*})q_1$$

Within each period, production takes place along a concave production possibilities frontier. It is assumed that output of each good depends only on that period's domestic relative price of the good and a productivity factor:

$$(2.1) \quad a_i^s = A_i f^a[p_i]; \quad \frac{df^a}{dp_i} < 0$$

$$(2.2) \quad n_i^s = N_i f^n[p_i]; \quad \frac{df^n}{dp_i} > 0$$

$$(2.3) \quad p_i = \frac{P_{ni}}{P_{ai}}$$

where:

a_i^s (n_i^s) = production of good a (good n) in period i;

p_i = domestic relative price of good n in period i

$$(p_i = P_{ni}/P_{ai});$$

P_{ai} (P_{ni}) = domestic nominal price of good a (good n) in i;

A_i (N_i) = multiplicative productivity term acting to shift

the supply of good a (good n).

And since the nation exports good n and imposes a quota on importing good a, the balance of trade measured in terms of the world price of good a (equations (2.29a) and (2.29b)) can be shown as:

$$(3.29a) \quad tb = p^* (n^s - n) - q_0$$

In addition, this general equilibrium model has two market clearing conditions. The demand for agricultural good in each period is equal to the supply of agricultural good in that period plus the agricultural import allowed by the quota in that period.

$$(3.17a) \quad a_0 = a_0^s + q_0$$

$$(3.17b) \quad a_1 = a_1^s + q_1$$

From the definition of the real interest rates described in chapter 2:

$$(2.6a) \quad (1 + r_a) = (1 + i) \left(\frac{P_a}{P_{al}} \right)$$

$$(2.6b) \quad (1 + r_n) = (1 + i) \left(\frac{P_n}{P_{nl}} \right)$$

These two identities could be reduced to obtain

$$(3.18) \quad \frac{1+r_a}{1+r_n} = \frac{p_1}{p}$$

Note that we have a system of 8 equations {(3.15a) - (3.15d), (3.17a) - (3.17b), (3.18), and (3.29a)} and endogenous variables (a_0 , a_1 , n_0 , n_1 , tb , p_0 , p_1 , and r_a).

To do the comparative static analysis of this general equilibrium model, we substitute equation (3.15a) into (3.17a) and obtain:

$$(3.19) \quad a^s + q_0 = \theta \left(\frac{1+\rho}{2+\rho} \right) \left[a^s + pn^s + \frac{a_1^s}{1+r_a} + \frac{pn_1^s}{1+r_n} + T_0 + \frac{T_1}{1+r_a} \right]$$

By using equations (2.4) and (3.16), equation (3.19) becomes

$$(3.20) \quad a^s + q_0 = \theta \left(\frac{1+\rho}{2+\rho} \right) \left[y + \frac{Y_1}{1+r_a} + \left(1 - \frac{p}{p^*} \right) q_0 + \frac{1}{1+r_a} \left(1 - \frac{p_1}{p_1^*} \right) q_1 \right]$$

With total differentiation of equation (3.20), we obtain

$$(3.21) \quad Af^{a'}dp + f^a dA = \theta \left(\frac{1+\rho}{2+\rho} \right) \left[n^s dp + f^a dA + pf^n dN + \frac{n_1^s}{1+r_a} dp_1 \right. \\ \left. + \frac{1}{1+r_a} (f^a dA_1 + p_1 f^n dN_1) - \frac{Y_1}{(1+r_a)^2} dr_a - \frac{q_0}{p^*} dp \right]$$

$$\begin{aligned}
& + \frac{P}{P^*} q_0 dp^* - \frac{q_1}{(1+r_a)^2} dr_a - \frac{q_1}{P_1^* (1+r_a)} dp_1 \\
& + \frac{P_1 q_1}{P_1^* (1+r_a)^2} (P_1^* dr_a + (1+r_a) dp_1^*)]
\end{aligned}$$

Substituting equation (3.15) into (3.17a) and (3.17b),

$$a^s + q_0 = \theta \left(\frac{1+\rho}{2+\rho} \right) w$$

$$a_1^s + q_1 = \theta \left[\frac{1+r_a}{2+\rho} \right] w$$

These two equations could be reduced to

$$(3.22) \quad (a^s + q_0) (1+r_a) = (a_1^s + q_1) (1+\rho)$$

Total differentiating equation (3.22),

$$(3.23) \quad (1+r_a) (A f^{a'} dp + f^a dA) + (a^s + q_0) dr_a = (1+\rho) (A_1 f^{a'} dp_1 + f^a dA_1)$$

Similarly, total differentiating equation (3.18) and rewrite it to obtain

$$(3.24) \quad (1+r_a) dp - (1+r_n) dp_1 + p dr_a = p_1 dr_n$$

Furthermore, total differentiating equations (3.15c), (3.15d), (3.29a), (3.17a), and (3.17b),

$$\begin{aligned}
 (3.25a) \quad dn = & \left(\frac{(1-\theta)(1+\rho)}{2+\rho} \right) \left[n^s dp + f^a dA + p f^n dN + \frac{n_1^s}{1+r_a} dp_1 \right. \\
 & + \frac{1}{1+r_a} (f^a dA_1 + p_1 f^n dN_1) - \frac{y_1}{(1+r_a)^2} dr_a - \frac{q_0}{p^*} dp \\
 & + \frac{p}{p^*} q_0 dp^* - \frac{q_1}{(1+r_a)^2} dr_a - \frac{q_1}{p_1^* (1+r_a)} dp_1 \\
 & \left. + \frac{p_1 q_1}{p_1^* (1+r_a)^2} (p_1^* dr_a + (1+r_a) dp_1^*) \right]
 \end{aligned}$$

$$\begin{aligned}
 (3.25b) \quad dn_1 = & \left[\frac{(1-\theta)(1+r_a)}{2+\rho} \right] \left[\frac{w}{(1+r_a)} dr_a + n^s dp + f^a dA + p f^n dN + \frac{n_1^s}{1+r_a} dp_1 \right. \\
 & + \frac{1}{1+r_a} (f^a dA_1 + p_1 f^n dN_1) - \frac{y_1}{(1+r_a)^2} dr_a - \frac{q_0}{p^*} dp \\
 & + \frac{p}{p^*} q_0 dp^* - \frac{q_1}{(1+r_a)^2} dr_a - \frac{q_1}{p_1^* (1+r_a)} dp_1 \\
 & \left. + \frac{p_1 q_1}{p_1^* (1+r_a)^2} (p_1^* dr_a + (1+r_a) dp_1^*) \right]
 \end{aligned}$$

$$(3.26) \quad d(tb) = p^* (N f^n dp + f^n dN - dn) + (n^s - n) dp^*$$

$$(3.27a) \quad da = A f^{a'} dp + f^a dA$$

$$(3.27b) \quad da_1 = A_1 f^{a'} dp_1 + f^a dA_1$$

Please note that equations (3.21), (3.23), and (3.24) form a system of three equations with three endogenous variables (p , p_1 , and r_s). Also, r_a is exogenous in this system. Adding equations (3.25a), (3.25b), (3.26), (3.27a), and (3.27b) to the system, we can rewrite the equation systems in a form of matrices as:

$$\begin{bmatrix}
\frac{(2+\rho)}{(1+\rho)\theta} Af^{a'} - n^s + \frac{q}{p^*} & -\frac{n_1^s}{1+I_a} + \frac{q_1}{p_1^*(1+I_a)} & \frac{y_1+T_1}{(1+I_a)^2} & 0 & 0 & 0 & 0 \\
(1+I_a) Af^{a'} & -(1+\rho) A_1 f^{a'} & a & 0 & 0 & 0 & 0 \\
(1+I_a) & -(1+I_n) & p & 0 & 0 & 0 & 0 \\
-n^s + \frac{q}{p^*} & -\frac{n_1^s}{(1+I_a)} + \frac{q_1}{p_1^*(1+I_a)} & \frac{y_1+T_1}{(1+I_a)^2} & \frac{(2+\rho)}{(1-\theta)(1+\rho)} & 0 & 0 & 0 \\
-n^s + \frac{q}{p^*} & -\frac{n_1^s}{(1+I_a)} + \frac{q_1}{p_1^*(1+I_a)} & \frac{y_1+T_1}{(1+I_a)^2} - \frac{w}{(1+I_a)} & \frac{(2+\rho)}{(1-\theta)(1+I_a)} & 0 & 0 & 0 \\
-p^* N f^{n'} & 0 & 0 & p^* & 0 & 1 & 0 \\
-A f^{a'} & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & -A_1 f^{a'} & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
dp \\
dp_1 \\
dr_a \\
dn \\
dn_1 \\
dtb \\
da \\
da_1
\end{bmatrix}$$

$$\begin{bmatrix}
\left(1 - \frac{(2+\rho)}{(1+\rho)\theta}\right) f^a dA + \frac{f^a}{(1+I_a)} dA_1 + p f^n dN + \frac{p_1 f^n}{(1+I_a)} dN_1 + \frac{pq}{p^{*2}} dp^* + \frac{p_1 q_1}{p_1^{*2}(1+I_a)} dp_1^* \\
(- (1+I_a) f^a) dA + ((1+\rho) f^a) dA_1 \\
p_1 dr_n \\
f^a dA + \frac{f^a}{(1+I_a)} dA_1 + p f^n dN + \frac{p_1 f^n}{(1+I_a)} dN_1 + \frac{pq}{p^{*2}} dp^* + \frac{p_1 q_1}{p_1^{*2}(1+I_a)} dp_1^* \\
f^a dA + \frac{f^a}{(1+I_a)} dA_1 + p f^n dN + \frac{p_1 f^n}{(1+I_a)} dN_1 + \frac{pq}{p^{*2}} dp^* + \frac{p_1 q_1}{p_1^{*2}(1+I_a)} dp_1^* \\
p^* f^n dN + (n^s - n) dp^* \\
f^a dA \\
f^a dA_1
\end{bmatrix}
=$$

For simplicity, these equations are evaluated at $p = p^*$ and $p_1 = p_1^*$. The followings are the summary of the relevant results.

$$\frac{dp}{dA} = -\frac{1}{\Delta} f^a \left(-\frac{p}{p_1} a_1^s + \left(1 - \frac{(2+p)}{(1+p)\theta} \right) ((1+r_n)(a^s+q) - (1+p)pA_1 f^{a'}) \right) < 0$$

$$\frac{dp}{dN} = \frac{1}{\Delta} p(1+r_n) f^n (a - pA_1 f^{a'}) < 0$$

$$\frac{dp}{dA_1} = \frac{1}{\Delta} \frac{1}{(1+r_a)} f^a \left((1+p) \frac{p}{p_1} a_1^s + (1+r_n)(a^s+q) - (1+p)pA_1 f^{a'} \right) < 0$$

$$\frac{dp}{dN_1} = \frac{1}{\Delta} \frac{p_1}{(1+r_a)} f^n ((1+r_n)(a - (1+p)pA_1 f^{a'}) < 0$$

$$\frac{dp}{dr_n^*} = \frac{1}{\Delta} \frac{p_1}{(1+r_a)} \left(-n_1^s a + \frac{(1+p)}{(1+r_a)} y_1 A_1 f^{a'} \right) > 0$$

$$\frac{dp}{dr_a^*} = \frac{\frac{1}{(1+r_a)} \left(-n_1^s a + \frac{(1+p)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{f^{a'} \left(A n_1^s + (1+p) A_1 n^s - \frac{(2+p)}{\theta} A A_1 f^{a'} \right)} > 0$$

$$\frac{dp_1}{dA} = -\frac{1}{\Delta} f^a \left(-\left(y_0 + \frac{y_1}{(1+r_a)} \right) - q + pA f^{a'} + \frac{(2+p)}{(1+p)\theta} a \right) < 0$$

$$\frac{dp_1}{dN} = \frac{1}{\Delta} p f^n (1+r_a) (a - pA f^{a'}) < 0$$

$$\frac{dp_1}{dA_1} = \frac{1}{\Delta} f^a \left(-(1+p) \left(y + \frac{y_1}{(1+r_a)} \right) + \left(\frac{(2+p)}{\theta} - 1 \right) pA f^{a'} + a + (1+p) a^s \right) = ?$$

$$\frac{dp_1}{dN_1} = \frac{1}{\Delta} p_1 f^n (a - p A f^{a'}) < 0$$

$$\frac{dp_1}{dp^*} = \frac{(1+r_a) \left(a \left(n^s - \frac{(2+p)}{(1+p)\theta} A f^{a'} \right) + \frac{A f^{a'} y_1}{(1+r_a)} \right)}{\left(-n_1^s a + \frac{(1+p)}{(1+r_a)} A_1 f^{a'} y_1 \right)} = ?$$

$$\frac{dp_1}{dr_n} = \frac{1}{\Delta} p_1 \left(\frac{(1+p)\theta}{(2+p)} n^{s_w-y} A f^{a'} \right) < 0$$

$$\frac{dp_1}{dr_a} = \frac{n^s a - \frac{(2+p)}{(1+p)\theta} A f^{a'} a + \frac{A f^{a'}}{(1+r_a)}}{f^{a'} \left(A n_1^s + (1+p) A_1 n^s - \frac{(2+p)}{\theta} A A_1 f^{a'} \right)} < 0$$

$$\begin{aligned} \frac{dr_a}{dA} &= \frac{1}{\Delta} \left[- (1+r_a) f^a \left(-n_1^s - (1+r_n) \left(n^s - \frac{(2+p)}{(1+p)\theta} A f^{a'} \right) \right) \right. \\ &\quad \left. - \left(1 - \frac{(2+p)}{(1+p)\theta} \right) f^a \left((1+r_a) \left((1+p) A_1 f^{a'} - (1+r_n) A f^{a'} \right) \right) \right] < 0 \end{aligned}$$

$$\frac{dr_a}{dN} = \frac{1}{\Delta} p f^n (1+r_a) f^{a'} (A_1 (1+p) - A (1+r_n)) = 0 \text{ if } A = A_1$$

$$\begin{aligned} \frac{dr_a}{dA_1} &= \frac{1}{\Delta} \left[(1+p) f^a \left(-n_1^s - (1+r_n) \left(n^s - \frac{(2+p)}{(1+p)\theta} A f^{a'} \right) \right) \right. \\ &\quad \left. + f^a \left((1+p) A_1 f^{a'} - (1+r_n) A f^{a'} \right) \right] > 0 \end{aligned}$$

$$\frac{dr_a}{dN_1} = \frac{1}{\Delta} p_1 f^{n f^{a'}} (A_1 (1+p) - A (1+r_n)) = 0 \text{ if } A = A_1$$

$$\frac{dr_a}{dr_n} = \frac{1}{\Delta} p_1 f^a \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho) A A_1 f^{a'}}{\theta} \right) > 0$$

$$\frac{dr_a}{dp} = \frac{(1+r_a) \left(A f^{a'} n_1^s + (1+\rho) A_1 f^{a'} n^s - \frac{(2+\rho) A f^{a'} A_1 f^{a'}}{\theta} \right)}{\left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)} > 0$$

$$\frac{da}{dA} = A f^{a'} \left(-\frac{1}{\Delta} f^a \left(-\frac{p}{p_1} a_1^s + \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) \left((1+r_n) (a^s + q) - (1+\rho) A_1 f^{a'} p \right) \right) + f^a \right) > 0$$

$$\frac{da}{dA_1} = A f^{a'} \frac{1}{\Delta} \frac{1}{(1+r_a)} f^a \left((1+\rho) \frac{p}{p_1} a_1^s + (1+r_n) (a^s + q) - (1+\rho) p A_1 f^{a'} \right) > 0$$

$$\frac{da}{dA} + \frac{da}{dA_1} = f^a + A f^{a'} \left(f^a \frac{1}{\Delta} \left(\left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \frac{(1+\theta + (1-\theta)(1+r_a))}{\theta(1+r_a)} \right) \right) > 0$$

$$\frac{da}{dN} = \frac{1}{\Delta} p (1+r_n) A f^{a'} f^n \left(a - \frac{(1+\rho)}{(1+r_n)} p A_1 f^{a'} \right) > 0$$

$$\frac{da}{dN_1} = \frac{1}{\Delta} \frac{p_1}{(1+r_a)} A f^{a'} f^n \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) > 0$$

$$\frac{da}{dN} + \frac{da}{dN_1} = \frac{1}{\Delta} f^n A f^{a'} \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \left(p + \frac{p_1}{(1+r_a)} \right) > 0$$

$$\frac{dn}{dA} = -\frac{1}{\Delta} \frac{(1-\theta)}{\theta} f^a \left[a (n_1^s + (1+r_n) n^s - (1+r_n) A f^{a'} - (1+\rho) A_1 f^{a'} \left(n^s + \frac{y_1}{(1+r_a)} \right) + (1+\rho) A A_1 p f^{a'/2} \right] > 0$$

$$\frac{dn}{dA_1} = \frac{1}{\Delta} \frac{(1+\rho)(1-\theta)}{(2+\rho)(1+r_a)} f^a \left[- \left(\frac{(2+\rho)}{\theta} - 1 \right) \frac{p}{p_1} A f^{a'} a_1^s \right. \\ \left. + a(n_1^s + n^s(1+r_n)) - (1+\rho) A_1 f^{a'} \left(p n^s + \frac{y_1}{(1+r_a)} \right) \right] < 0$$

$$\frac{dn}{dA} + \frac{dn}{dA_1} = \frac{1}{\Delta} (1-\theta) f^a \left[\left(\frac{\theta(1+\rho) - (2+\rho)(1+r_a)}{\theta(2+\rho)(1+r_a)} \right) a(n_1^s + (1+r_n)n^s) \right. \\ \left. - \left(\frac{\theta(1+\rho) - (2+\rho)(1+r_a)}{\theta(2+\rho)(1+r_a)} \right) A_1 f^{a'} \left(n^s p + \frac{y_1}{(1+r_a)} \right) \right. \\ \left. - A f^{a'} \left[\frac{(1+\rho)}{\theta} p A_1 f^{a'} + \frac{(1+\rho)}{(1+r_a)} \left(\frac{1}{\theta} - \frac{1}{(2+\rho)} \right) \frac{p}{p_1} a_1^s \right. \right. \\ \left. \left. - \frac{(1+r_n)}{\theta} a \right] \right] > 0$$

$$\frac{dn}{dN} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} A f^{a'} ((1+r_n)a - (1+\rho)p A_1 f^{a'}) > 0$$

$$\frac{dn}{dN_1} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta(1+r_a)} p_1 f^n A f^{a'} ((1+r_n)a - (1+\rho)p A_1 f^{a'}) > 0$$

$$\frac{dn}{dN} + \frac{dn}{dN_1} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} A f^{a'} ((1+r_n)a - (1+\rho)p A_1 f^{a'}) \left(1 + \frac{p_1}{(1+r_a)} f^n \right) > 0$$

$$\frac{dtb}{dA} = \frac{1}{\Delta} p^* f^a \left[N f^n \left(\frac{p}{p_1} a_1^s - \left(1 - \frac{(2+\rho)}{(1+\rho)\theta} \right) ((1+r_n)a - (1+\rho)p A_1 f^{a'}) \right) \right. \\ \left. + \frac{(1-\theta)}{\theta} [a(n_1^s + (1+r_n)n^s - (1+r_n)A f^{a'}) \right. \\ \left. - (1+\rho) f^a A_1 f^{a'} \left(p n^s + \frac{y_1}{(1+r_a)} \right) + (1+\rho) p A A_1 f^{a/2}] \right] < 0$$

$$\frac{dtb}{dA_1} = \frac{1}{\Delta} \frac{p^* f^a}{(1+r_a)} \left[\frac{p}{p_1} a_1^s \left((1+\rho) N f^{n'} + \left(\frac{(2+\rho)}{\theta} - 1 \right) A f^{a'} \right) \right. \\ \left. + a((1+r_n) N f^{n'} - (n_1^s + (1+r_n)n^s)) \right. \\ \left. + (1+\rho) A_1 f^{a'} \left(-p N f^{n'} + \left(n^s p + \frac{y_1}{(1+r_a)} \right) \right) \right] = ?$$

$$\begin{aligned} \frac{dtb}{dA} + \frac{dtb}{dA_1} &= \frac{1}{\Delta} p^* f^a \left[Nf^n \left(\frac{(1+r_n)}{(1+r_a)} a + \left(1 + \frac{(1+\rho)}{(1+r_a)} \right) \frac{p}{p_1} a_1^s \right) \right. \\ &\quad \left. - \frac{(1-\theta)}{\theta} (1+r_n) A f^{a'} a \right. \\ &\quad \left. + (1+\rho) p A_1 f^a \left(-\frac{1}{(1+r_a)} Nf^{n'} + \frac{(1-\theta)}{\theta} A f^{a'} \right) \right. \\ &\quad \left. + (1+\rho) A_1 f^a \left(p n^s + \frac{y_1}{(1+r_a)} \right) \left(\frac{1}{(1+r_a)} - \frac{(1-\theta)}{\theta} f^a \right) \right. \\ &\quad \left. + a (n_1^s + (1+r_n) n^s) \left(-\frac{1}{(1+r_a)} + \frac{(1-\theta)}{\theta} \right) \right] = ? \end{aligned}$$

$$\frac{dtb}{dN} = p^* \left(f^n + \frac{1}{\Delta} \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \left(Nf^{n'} f^n p - \frac{(1-\theta)}{\theta} A f^{a'} \right) \right) = ?$$

$$\frac{dtb}{dN_1} = \frac{1}{\Delta} p^* \frac{p_1}{(1+r_a)} f^n \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \left(Nf^{n'} - \frac{(1-\theta)}{\theta} A f^{a'} \right) < 0$$

$$\begin{aligned} \frac{d(tb)}{dN} + \frac{d(tb)}{dN_1} &= p^* f^n + \frac{1}{\Delta} \left((1+r_n) a - (1+\rho) p A_1 f^{a'} \right) \left[p^* f^n Nf^n \left(p + \frac{p_1}{(1+r_a)} \right) \right. \\ &\quad \left. - \frac{(1-\theta)}{\theta} p^* A f^a \left(1 + \frac{p_1}{(1+r_a)} f^n \right) \right] = ? \end{aligned}$$

$$\frac{da_1}{dA} = -\frac{1}{\Delta} (1+r_a) f^a A_1 f^{a'} \left(-w - q_0 + \frac{(2+\rho)}{\theta(1+\rho)} a \right) > 0$$

$$\frac{da_1}{dA_1} = \frac{1}{\Delta} A_1 f^{a'} f^a \left(-(1+\rho) \left(y + \frac{y_1}{(1+r_a)} \right) + \left(\frac{(2+\rho)}{\theta} - 1 \right) p A f^{a'} + a + (1+\rho) a^s \right) + f^a = ?$$

$$\begin{aligned} \frac{da_1}{dA} + \frac{da_1}{dA_1} &= A_1 f^{a'} \left[-\frac{1}{\Delta} f^a \left(-w - q + \frac{(2+\rho)}{(1+\rho)\theta} a \right) \right. \\ &\quad \left. + \frac{1}{\Delta} f^a \left[-(1+\rho) \left(p n^s + \frac{y_1}{(1+r_a)} \right) \right. \right. \\ &\quad \left. \left. + \left(\frac{(2+\rho)}{\theta} - 1 \right) p A f^{a'} + a \right] \right] + f^a = ? \end{aligned}$$

$$\frac{da_1}{dN} = \frac{1}{\Delta} (1+r_a) p A_1 f^{a'} f^n (a - p A f^{a'}) > 0$$

$$\frac{da_1}{dN_1} = \frac{1}{\Delta} p_1 f^n A_1 f^{a'} (a - p A f^{a'}) > 0$$

$$\frac{da_1}{dN} + \frac{da_1}{dN_1} = A_1 f^{a'} \left(\frac{1}{\Delta} p f^n (1+r_a) (a - p A f^{a'}) + \frac{1}{\Delta} p_1 f^n (a - p A f^{a'}) \right) > 0$$

$$\begin{aligned} \frac{dn_1}{dA} = & -\frac{1}{\Delta} \frac{(1-\theta)(1+r_a)}{\theta(1+\rho)} f^a [a(n_1^s + (1+r_n)n^s - (1+r_n)A f^{a'}) \\ & - (1+\rho)A_1 f^a \left(n^s + \frac{y_1}{(1+r_a)} \right) + (1+\rho)AA_1 p f^{a/2}] > 0 \end{aligned}$$

$$\begin{aligned} \frac{dn_1}{dA_1} = & \frac{1}{\Delta} \frac{(1-\theta)}{(2+\rho)} f^a \left[-\left(\frac{(2+\rho)}{\theta} - 1 \right) \frac{p}{p_1} A f^{a'} a_1^s + a(n_1^s + n^s(1+r_n)) \right. \\ & \left. - (1+\rho)A_1 f^{a'} \left(p n^s + \frac{y_1}{(1+r_a)} \right) \right] < 0 \end{aligned}$$

$$\begin{aligned} \frac{dn_1}{dA} + \frac{dn_1}{dA_1} = & \frac{1}{\Delta} (1-\theta) f^a \left[\left(\frac{\theta(1+\rho) - (2+\rho)(1+r_a)}{\theta(2+\rho)(1+\rho)} \right) a(n_1^s + (1+r_n)n^s) \right. \\ & - \left(\frac{\theta(1+\rho) - (2+\rho)(1+r_a)}{\theta(2+\rho)} \right) A_1 f^{a'} \left(n^s p + \frac{y_1}{(1+r_a)} \right) \\ & - A f^{a'} \left[\frac{(1+r_a)}{\theta} p A_1 f^{a'} + \left(\frac{(2+\rho) - \theta}{\theta(2+\rho)} \right) \frac{p}{p_1} a_1^s \right. \\ & \left. \left. - \frac{(1+r_n)(1+r_a)}{\theta(1+\rho)} a \right] \right] > 0 \end{aligned}$$

$$\frac{dn_1}{dN} = \frac{1}{\Delta} \frac{(1-\theta)(1+r_a)}{\theta} A f^a \left(\frac{(1+r_n)}{(1+\rho)} a - p A_1 f^{a'} \right) > 0$$

$$\frac{dn_1}{dN_1} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} p_1 f^n A f^a \left(\frac{(1+r_n)}{(1+\rho)} a - p A_1 f^{a'} \right) > 0$$

$$\frac{dn_1}{dN} + \frac{dn_1}{dN_1} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} A f^a \left(\frac{(1+r_n)}{(1+\rho)} a - p A_1 f^{a'} \right) ((1+r_a) + p_1 f^n) > 0$$

$$\frac{da}{dp^*} = Af^{a'} < 0$$

$$\frac{da}{dr_n^*} = \frac{1}{\Delta} \frac{P_1}{(1+r_a)} Af^{a'} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} y_1 A_1 f^{a'} \right) < 0$$

$$\frac{da}{dr_a^*} = Af^{a'} \frac{\frac{1}{(1+r_a)} \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} < 0$$

$$\frac{dn}{dp^*} = \frac{(1-\theta)}{\theta} Af^{a'} < 0$$

$$\frac{dn}{dr_n^*} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta (1+r_a)} P_1 Af^{a'} \left(-n_1^s a + (1+\rho) A_1 f^{a'} \frac{y_1}{(1+r_a)} \right) < 0$$

$$\frac{dn}{dr_a^*} = \frac{(1-\theta)}{(1+r_a) \theta} \frac{A \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'}} < 0$$

$$\frac{dtb}{dp^*} = (n^s - n) - p^* \left(\frac{(1-\theta)}{\theta} Af^{a'} - N f^{n'} \right) > 0$$

$$\frac{dtb}{dr_n^*} = \frac{1}{\Delta} P_1 \left(\left(\frac{p^*}{(1+r_a)} n_1^s a - A_1 f^{a'} \frac{(1+\rho)}{(1+r_a)^2} p^* y_1 \right) \left(\frac{(1-\theta)}{\theta} Af^{a'} - N f^{n'} \right) \right) > 0$$

$$\frac{dtb}{dr_a^*} = \frac{(1-\theta)}{(1+r_a)} \frac{p^* \left(n_1^s a - \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right) \left(\frac{Af^{a'}}{\theta} - \frac{N f^{n'}}{(1-\theta)} \right)}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} > 0$$

$$\frac{da_1}{dp^*} = A_1 f^{a'} \frac{(1+r_a) \left(a \left(n^s - \frac{(2+\rho)}{(1+\rho)\theta} A f^{a'} \right) + \frac{A f^{a'} y_1}{(1+r_a)} \right)}{\left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)} < 0$$

$$\frac{da_1}{dr_n^*} = \frac{1}{\Delta} p_1 A_1 f^{a'} \left(\frac{(1+\rho)\theta}{(2+\rho)} n^s w - y A f^{a'} \right) > 0$$

$$\frac{da_1}{dr_a^*} = A_1 f^{a'} \frac{n^s a - \frac{(2+\rho)}{(1+\rho)\theta} A f^{a'} a + \frac{A f^{a'}}{(1+r_a)}}{f^{a'} \left(A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'} \right)} > 0$$

$$\frac{dn_1}{dp^*} = \frac{(1-\theta)}{(1+\rho)\theta} (1+r_a) A f^{a'} < 0$$

$$\frac{dn_1}{dr_n^*} = \frac{1}{\Delta} \frac{(1-\theta)}{\theta} p_1 A f^{a'} \left(-\frac{n_1^s a}{(1+\rho)} + A_1 f^{a'} \frac{y_1}{(1+r_a)} \right) < 0$$

$$\frac{dn_1}{dr_a^*} = \frac{(1-\theta)}{(1+\rho)\theta} \frac{A \left(-n_1^s a + \frac{(1+\rho)}{(1+r_a)} A_1 f^{a'} y_1 \right)}{A n_1^s + (1+\rho) A_1 n^s - \frac{(2+\rho)}{\theta} A A_1 f^{a'}} < 0$$